Now we come to:

**Centripetal Acceleration**

(centr- seeking acc)

To do with "point objects" executing *circular motion*

E.G.: Earth around Sun

Tennis ball fixed onto string

Car going over "hump backed" bridge
**Circular Motion at Constant Speed:**

\[ \mathbf{v} = \text{const} \]

**But \( \mathbf{v} \neq \text{const} \)**

\[ \Delta \mathbf{v} = \mathbf{v}_B - \mathbf{v}_A \]

**\( \Delta \mathbf{v} \):**

\[ \begin{align*}
\frac{\mathbf{v}_B}{\mathbf{v}_A} & \to \downarrow \\
\Delta \mathbf{v} & \to \\
\overrightarrow{\Delta \mathbf{v}} &
\end{align*} \]

**Move B close to A:**

\[ \Delta \mathbf{v} \]

\[ \overrightarrow{\mathbf{a}} \text{ towards center.} \]
LET'S GO BACK TO SCALARS.

WE WILL FIND THE ACC. TOWARDS THE CENTER:

CENTRIPETAL ACC. $a_c$:

$$a_c = \frac{v^2}{r}$$

AND FROM NEWTON # 2 THE FORCE REQUIRED TO RESULT IN CIRCULAR MOTION IS

$$F_c = ma_c = m\frac{v^2}{r}$$

(Book uses vectors)
My simplified scalar proof using components

Consider circular motion in horizontal plane (x-y) with constant speed u

Step 1: Consider the instant that the object crosses the x-axis:

\[ F_y = 0 \] at this instant (ie a_y = 0)

So the only force possible is

\[ F_x \neq 0 \] at this instant

ie towards the center of the circle
Step 2: A short time \( t \) later the object has swept out angle \( \Delta \theta \):

\[
\begin{align*}
\vec{u}_1 & \rightarrow \frac{\vec{u}_x}{\Delta \theta} \\
\vec{u}_1 & \rightarrow \vec{u}_x \\
\vec{u}_x & \rightarrow \Delta \theta \\
\vec{u}_x & \rightarrow \text{Radius } R \\
\end{align*}
\]

\[
\begin{align*}
\vec{u}_x & \rightarrow 0 \\
\vec{u}_x & \rightarrow \frac{u \cos (90 - \Delta \theta)}{\Delta t} = u \sin \Delta \theta \\
\end{align*}
\]

\[
\begin{align*}
\overrightarrow{a}_x & = \frac{\Delta \vec{u}_x}{\Delta t} = \frac{\vec{u}_x - \vec{u}_x}{\Delta t} = \frac{u \sin \Delta \theta}{\Delta t} \\
\end{align*}
\]

As \( \Delta \theta \rightarrow \text{smaller and smaller} \)

\[
\begin{align*}
\sin \Delta \theta & \rightarrow \Delta \theta \quad [\text{in radians}] \\
\end{align*}
\]

Check:

\[
\begin{array}{c|c|c}
\theta & \sin \theta & 0.8415 \\
0.1 & 0.0998 & 0.009998 \\
0.01 & 0.009998 & \\
\end{array}
\]
so \[ a_x = v \frac{\Delta \theta}{\Delta t} = v \omega \]

**USE** \[ \omega = \frac{v}{r} \]

\[ a_x = v \frac{v}{r} = \frac{v^2}{r} \]

---

**BUT THERE IS NOTHING SPECIAL ABOUT OUR POINT.**

so:

\[ a_c = \frac{v^2}{r} \]

or

\[ a_c = \omega^2 r \]

since \[ v = \omega r \] ; \[ \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r \]
Newton #2 is always true.

So if

\[ \alpha_c = \frac{v^2}{r} = \omega^2 + \]

there must exist a net force

\[ F_c = \frac{mv^2}{r} = mw^2 \]

that is causing the acceleration.
Let's examine our three examples:

1) Earth around Sun

2) Ball whirled around on end of string

3) Car on hump back bridge

Important:

Use free body diagrams!

$F_c = rac{mv^2}{r}$ is required magnitude of force due to

1. Gravitational force
2. Tension
3. Wait and see ....
1) Earth around Sun.

Due to gravitational attraction

\[ F = \frac{G M E M S}{r^2} \]

\[ F_c = M_E \frac{u^2}{r} \]

So

\[ F_c = F_c \]

\[ \frac{G M_E M_S}{r^2} = M_E \frac{u^2}{r} \]

OR

\[ \frac{u^2}{r} = \frac{G M_S}{r} \]

So all planets have

\[ \frac{u^2}{r} = \frac{G M_S}{r} \]

(Circular motion ...)
2) **BALL ON END OF STRING**

![Diagram of a ball on an end of a string with forces labeled]

**FBD:**

\[ T = F_c \]

So, \[ T = \frac{m u^2}{r} \]

DONE!

(Note: horizontal plane and ignore gravity)
3) Car on Bridge radius r

Assume $v \neq \text{const}.$

Two physical forces:

- Gravity
- Normal force

FBD:

\[
\begin{align*}
\text{mg} & \quad \theta \\
& \quad n \\
& \quad \Rightarrow \\
& \quad mg \sin \theta \\
& \quad mg \cos \theta
\end{align*}
\]

So $F_c = \frac{mv^2}{r} = mg \cos \theta - n$

At any instant

(We will do a worked problem later)
COMMUNICATIONS SATELLITES IN GEO STATIONARY ORBITS

For such an orbit the satellite is always directly above the same point of the Earth.

\[ \omega' = \omega \]

Also the orbital period is 1 day!

\[ T = 24 \text{ hours} \]

We can find the conditions on \( r \) (orbital radius) and \( v \) (velocity).
13

**MASS OF EARTH**

\[ \text{Earth} \]

\[ m_{\text{SAT}} = \frac{M}{m} \]

**EARTH ATTRACTION SAT.**

\[ F_c = G \frac{m_1 m_2}{r^2} \]

**CIRCULAR MOTION:**

\[ F_c = m \frac{v^2}{r} \]

\[ \frac{m v^2}{r} = G \frac{M}{r^2} \]

\[ \frac{v^2 r}{r} = G M \]

**ORBITAL PERIOD**

\[ T = \frac{\text{ORBIT}}{\text{SPEED}} \]

\[ v = \frac{2 \pi r}{T} \]

\[ \Rightarrow \]

**PUT (2) INTO (1):**

\[ \frac{(2 \pi)^2 r^3}{T^2} = G M \]
Solve for $r$:

$$r = \left( \frac{GMT^2}{4\pi^2} \right)^{\frac{1}{3}}$$

And speed:

$$v = \frac{2\pi r}{T}$$

$T = 24\text{ h}$

**Putting in numbers:**

A geostationary satellite orbit at

$$r = 42,300\text{ km}$$

At

$$v = 3.08\text{ km/s}$$

[Note $r = 22,000\text{ m}$ above surface of Earth]

[SiriusXM radio < Free AD]
SO TO SUMMARIZE:

CIRCULAR MOTION REQUIRES A CENTER-SEEKING FORCE OF MAGNITUDE

\[ F_c = \frac{m u^2}{r} \]

THIS EQUATION DOES NOT TELL YOU THE ORIGIN OF THE FORCE — IT MERELY TELLS YOU WHAT MAGNITUDE IS REQUIRED FOR A GIVEN \( m, u, r \).

THE ORIGIN OF THE FORCE DEPENDS ON THE PHYSICAL SITUATION.

IT MIGHT BE GRAVITY, TENSION, OR EVEN THE NORMAL FORCE — (SEE LAST WORKED EXAMPLE ON SCHEDULE)
WE DO NOT DO (CH-7)

HEPLER'S LAWS

WE DID THE SEO-SATIONARY CASE WITHOUT THEM.

Finally.