Newton's 3rd Law

So far we've discussed forces acting on one body:

\[ F_{mg} \]

Newton #3 to do with 2 bodies:

\[ F_{AB} \text{ (due to A on B)} \] equal and opposite to \[ F_{BA} \text{ (due to B on A)} \]

Or

\[ F_{AB} = -F_{BA} \]

Or

To every action there is an equal and opposite reaction

Or

Action and reaction are equal and opposite
N#3: BE CAREFUL!

There must be two bodies

so block on table is

two bodies but when we

draw an FBD,

N and mg both act on

the block.

They are not the

action and reaction

forces (even though they

are equal and opposite!)
Block on Table:

Two objects:

1) The block.

2) The table, or better, the Earth:

(Not to scale!)

Let's do this step by step.
Step 1 is to introduce Newton's law of gravity.

Take two point objects of mass $M_A$ and $M_B$.

Put them apart.

Newton:

Attractive gravitational force between them:

$$F = G \frac{M_A M_B}{R_{AB}^2}$$

"Big G" universal gravitational constant (experiment)
Also applies for spheres if $R_{AB}$ between centers.

(NEWTON INVENTED THE CALCULUS TO PROVE IT!)

FBD's:

A: $F = G \frac{M_A M_B}{R_{AB}^2}$

B: $F = G \frac{M_A M_B}{R_{AB}^2}$

Notice the forces are equal and opposite.
So gravity between two planets

(Note: $F_{BA}$ acts on $A$, $F_{AB}$ acts on $B$)

$F_{BA} = -F_{AB}$

Action and reaction pair

(Doesn't matter which is which)

In Newton's law can switch $A$ and $B$.

$F = G \frac{M_A M_B}{R_{AB}^2} = G \frac{M_B M_A}{R_{BA}^2}$
ACCELERATION OF A:

\[ a_A = \frac{F_{BA}}{M_A} \]

ACCELERATION OF B:

\[ a_B = \frac{F_{AB}}{M_B} \quad (= \frac{F_{BA}}{M_B}) \]

So in general

\[ F_{AB} = F_{BA} \]

But

\[ a_A \neq a_B \]

Unless \( M_A = M_B \)
Now let's assume the planets attract each other (or is it one another?) and move towards each other (or ...) until they touch and come to rest.

There are now (in addition to gravity) two normal forces $n_{AB}$ and $n_{BA}$.

$F_{BA}$ is the "contact" force on B due to A.

$F_{AB}$ is the "contact" force on A due to B.

Action reaction pairs: $F_{BA}, F_{AB}$ and $n_{BA}, n_{AB}$.
BUT WAIT THERE'S MORE:

THE SYSTEM IS AT REST

SO \( \alpha_A = 0 \) \( \alpha_B = 0 \)

DRAW `FBD's`

\[ n_{BA} \leftarrow F_{BA} \]

\[ F_{AB} \leftarrow n_{AB} \]

\[ \alpha_A = 0 \]
\[ n_{BA} = F_{BA} \]

\[ \alpha_B = 0 \]
\[ n_{AB} = F_{AB} \]

\[ n_{BA} = n_{AB} \]

AND

\[ F_{BA} = F_{AB} \]

SO HERE:

\[ F_{AB} = n_{AB} = F_{BA} = n_{BA} \]

BUT \( F_{AB} \) AND \( n_{AB} \) ARE NOT

ACTION REACTION PAIR

(NOR IS \( F_{BA} \) AND \( n_{BA} \))
I've said nothing about size or mass. Everything true regardless of size or mass.

$\mathbf{F}_{BA} = -\mathbf{F}_{AB}$

And when they touch we have action-reaction pairs as before.

$\mathbf{n}_{BA} \leftrightarrow \mathbf{n}_{AB}$

$\mathbf{F}_{BA} \leftrightarrow \mathbf{F}_{AB}$
Let "Planet B" be a block of wood! Planet A = Earth

Rotate picture:

\[ F_{EB} = G \frac{m_B m_E}{R_E^2} = m_B g \]

\[(so \ g = G \frac{M_E}{R_E^2})\]

**Note:** \( F_{EB} = F_{BE} \)

So block attracts Earth with same force as Earth attracts block!

**But** \( a_B = g \)

\[
a_E = \frac{F_{BE}}{m_E} = \frac{F_{EB}}{m_E} = \left(\frac{m_B}{M_E}\right)g
\]

\[\approx 0 \text{ V. small} \]

= Tiny!
Block resting on Earth:

Action and reaction pairs:

\[ \text{F}_\text{BE} \uparrow, \text{F}_\text{EB} \downarrow \]

\[ \text{n}_\text{BE} \downarrow, \text{n}_\text{EB} \uparrow \]

But FBD of block:

So \[ \text{F}_\text{BE} = \text{n}_\text{BE} = \text{n}_\text{EB} = m \text{B}g \]
Let's look at a fairly complicated example, and see if #3 is true!

**Push 2 blocks along table (no friction)**

\[ F_0 \rightarrow m_1 \parallel m_2 \]

\[ m_1 \neq m_2 \]

What is the net force on \( m_2 \) and \( m_1 \)?

**Use #2 on combined mass**

\[ \boxed{m_1 \parallel m_2 = \boxed{M}} \]

\[ M = m_1 + m_2 \]

\[ F_0 = Ma = (m_1 + m_2) \alpha \]

So \[ \alpha = F_0 / (m_1 + m_2) \]

Same for both blocks
USE \( F = ma \) ON EACH BLOCK

**NET \( F \) on \( m_1 \):** \( F_1 = m_1a = \frac{m_1}{m_1+m_2} \)

**NET \( F \) on \( m_2 \):** \( F_2 = m_2a = \frac{m_2}{m_1+m_2} \)

So each block feels a different net force!

**Counter-intuitive!**
(I’ll bet most would think \( F_1 = F_2 = F_0 \))

So what’s the “Physics” behind this problem?

Where does \( F_2 \) come from?

**Answer:** Normal Force!
Let's draw FBD's:

\[ \begin{align*}
\vec{F}_0 & \rightarrow [m] \leftarrow n_{21} \\
\rightarrow [m_2] & \rightarrow n_{12} 
\end{align*} \]

\( m_2 \) is pushing back on \( m_1 \)

\( m_1 \) is "forwards on \( m_2 \)

\#3 claims \( n_{21} = n_{12} \)

Is it? Let's see:

FBD's:

\[ \begin{align*}
F_1 &= F_0 - n_{21} \\
F_2 &= n_{12}
\end{align*} \]

\[ 
\begin{align*}
F_1 &= F_0 - m_2 F_0 = F_0 - F_2 \\
&= F_0 - \frac{m_2}{m_1 + m_2} F_0 \\
&= \frac{m_1 - m_2}{m_1 + m_2} F_0
\end{align*} \]

So

\[ F_1 = \frac{m_1}{m_1 + m_2} F_0 \]

which is what we got before.

So \( n_{12} = n_{21} \) is true.
The case of the fly and the pickup truck

Splat! Forces are involved.

Force of fly on truck \( F_{FT} \)

\[ F_{FT} = F_{TF} \]

But \( \alpha_F = \frac{F_{TF}}{m_F} \)

Big! (squashed fly)

\( \alpha_T = \frac{F_{FT}}{M_T} \)

Tiny! (truck unaffected)