Friction and Rolling

Friction: A force between two objects that are touching that opposes their motion.

Consider me pushing on an object on a table ($F_{\text{applied}}$). A frictional force opposes the applied force (e.g. me pushing on the object). It has a maximum value – if I keep pushing harder, eventually the object will move. This maximum value depends on how smooth the surfaces are but is also proportional to the normal force – i.e. I would have to push heavier objects harder to get them to move.

This frictional force, when the object is not moving is called static friction.
The static frictional force is because the surfaces are not perfectly smooth, so only touch at small points. The normal force between the surfaces is magnified at the points and tends to “weld” them together. It takes a force parallel to the surfaces larger than the maximum static frictional force to break these welds and get the objects to move.

Once the object is moving, there is still a frictional force, also proportional to the normal force, because all the surface jags tend to catch on each other. This is called “sliding friction”.
The sliding friction force is less than the maximum static friction force (the “break-away force”). You can observe this because the force you need to apply to keep the object moving at a constant speed is less than the force you needed to apply to get it started.

Since the object isn’t moving for static friction, static friction does no work. However sliding friction does do work: since the sliding frictional force is in the opposite direction as the displacement, the work it does is negative: \[ W_{\text{friction}} = -F_{\text{sliding friction}} \cdot d \], where \( d \) is the displacement.
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**Analysis:** The object has initial kinetic energy (= ½ Mv_i^2) and zero final kinetic energy. Therefore, the frictional force must have done work to remove this kinetic energy:

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W_{\text{friction}} = -F_{\text{friction}} \cdot d = \Delta (\text{Kinetic Energy}) = KE(\text{final}) - KE(\text{initial}) = 0 - \frac{1}{2} Mv_i^2
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F_{\text{friction}} = \frac{\frac{1}{2} Mv_i^2}{d} = \frac{\frac{1}{2} (7 \text{ kg}) (5 \text{ m/s})^2}{3 \text{ m}} = 29.2 \text{ kg m/s}^2 = 29.2 \text{ N}
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**Question:** The initial kinetic energy $= \frac{1}{2} Mv_i^2 = 87.5 \text{ J}$. Since energy is conserved, where did this energy go?

It went into the vibrational energy of all the atoms and molecules on the two surfaces that were rubbing. The atoms and molecules start vibrate harder because of the rubbing between the surfaces. We measure that increase in their energy as their temperature. Sliding friction has converted the kinetic energy of the sliding object to thermal energy of the atoms and molecules.
Sliding friction always turns “ordered” energy – i.e. for the sliding block, all of its atoms and molecules were moving together to give it its initial velocity and kinetic energy – to “disordered”, i.e. thermal energy – all the atoms and molecules jiggling independently. We will see when we study thermodynamics that it is easy to turn ordered energy into disordered but very difficult to turn disordered energy back into ordered energy. (This will be contained in the Second Law of Thermodynamics.)
How to avoid wasting energy through sliding friction: Put it on wheels and let them roll! (Rolling requires static friction to operate – otherwise the wheels will slide (i.e. skid).

The static\* frictional force, acting on the lever arm $r$, will put a torque $\tau = rF_{\text{static friction}}$ on the wheel (around its hub). The direction of the torque is out of the page, i.e. causing counter-clockwise turning.

The cart will travel distance $2\pi r$ when the wheel turns once, so

$$v = \omega r.$$ 

Once the wheel is turning at this speed, no further torque (and no friction) is needed. However torque (and static friction) are needed to change $\omega$.

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\[ \mathbf{v} \text{ (of center with respect to ground)} \]

\[ \mathbf{v} \text{ (of point on bottom with respect to center (i.e. in center’s reference frame))} \]

The velocity of point on bottom with respect to ground

\[ = \text{velocity of center with respect to ground} + \text{velocity of bottom with respect to center} \]

\[ = 0 \]
If the wheels are perfectly round and the surfaces aren’t bumpy, so the wheels can turn smoothly, and there is not much friction at the wheel hubs (e.g. use roller bearings) and if the air resistance is negligible, the object on wheels will roll along forever, i.e. it will keep its initial kinetic energy. Note that some of this kinetic energy is contained by the turning wheels:

\[ KE = KE \text{ of cart} + KE \text{ of wheels} = KE \text{ of translation} + KE \text{ of rotation} \]
\[ = \frac{1}{2} Mv^2 + \sum \frac{1}{2} I \omega^2 \]

Recall: \( I = mr^2 f \), where \( m \) and \( r \) are the mass and radius of the wheel, and \( f \) describes how its mass is distributed. Therefore
\[ \frac{1}{2} I \omega^2 = \frac{1}{2} m(r\omega)^2 f = \frac{1}{2} mv^2 f, \]
so the rotational kinetic energy will be much less than the translational kinetic energy if \( m \ll M \). That is, you will use relatively little energy in getting the wheels turning.
What about a rolling object – i.e. an object where all the mass is in the “wheel”?

KE = \( \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 \)

= \( \frac{1}{2} [Mv^2 + M(R\omega)^2 f] \)

= \( \frac{1}{2} M v^2 [1 + f] \)

Recall: \( f = 1 \) if all the mass is in the rim, i.e. hollow cylinder.
\( f = 0 \) if all the mass is in the center.
Other shapes: \( 0 < f < 1 \).
Consider a round object rolling down a ramp of height $h$. Initially, all its energy is gravitational potential energy $= Mgh$. At the bottom, all of its energy is kinetic $= \frac{1}{2} Mv^2 (1+f)$. If it rolls and does not skid, none of its energy is wasted and turns to heat, so $Mgh = \frac{1}{2} Mv^2 (1+f)$.

Final speed $v = \sqrt{\frac{2gh}{1+f}}$

Consider a race between a hollow cylinder, solid cylinder, hollow sphere, solid sphere. Who will get to the bottom first? Who last?
Exercises:
24. Professional sprinters wear spikes on their shoes to prevent them from sliding on the track at the start of a race. Why is energy wasted whenever a sprinter’s foot slides backward along the track?

25. A yo-yo is a spool-shaped toy that spins on a string. In a sophisticated yo-yo, the end of the string forms a loop around the yo-yo’s central rod so that the yo-yo can spin almost freely at the end of the string. Why does the yo-yo spin longest if the central rod is very thin and very slippery?

26. As you begin pedaling your bicycle and it accelerates forward, what is exerting the forward force that the bicycle needs to accelerate?

28. If you are pulling a sled along a level field at constant velocity, how does the force you are exerting on the sled compare to the force of sliding friction on its runners?

29. Why does putting sand in the trunk of a car help to keep the rear wheels from skidding on an icy road?

30. When you’re driving on a level road and ice is on the pavement, you hardly notice that ice while you’re heading straight at a constant speed. Why is it that you notice how slippery the road is only when you try to turn left or right, or to speed up or slow down?

Problems:
• An object with a weight = 120 N is sliding on a table. It’s initial speed = 2 m/s, and it slides for 1.5 meters before coming to rest? How large was the sliding frictional force?
• A hollow cylinder rolls down a ramp, so that its change in height = 0.5 m. If the cylinder started from rest, what is its speed at the bottom?