Correlations Calculation

Hopefully Finally There !

Correlation Calculation : Through Diagonalization

The Recipe :

1. Calculate the physics asymmetry :
For a run, calculate the physics asymmetry between two pulses of any wire. The yield in the asymmetry has to be sum over detectors normalized and pedestal subtracted.

$$A_i = \frac{1}{G_i} \frac{Y_i^{\uparrow} - Y_i^{\downarrow}}{Y_i^{\uparrow} + Y_i^{\downarrow}} \tag{22}$$

2. Calculate the covariance out of physics asymmetry :

$$C_{ij} = Cov(A^i, A^j) = \frac{1}{N} \sum_{k=1}^{N} (A_k^i - \bar{A}^i)(A_k^j - \bar{A}^j) \tag{23}$$

So C_{ij} is a 144x144 matrix and by construction its a symmetric matrix.

3. Diagonalize the covariance matrix :
Noting the fact that matrix C is symmetric, find out a matrix S such that –

$$S^T C S = D \tag{24}$$

Where D is a diagonal matrix wit diagonal elements

$$D = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2 ..., \sigma_{144}^2) \tag{25}$$

The matrix S is the transformation matrix to be used to transform back and forth between the correlated and uncorrelated basis(reference frame).
The matrix S can be built up from the eigenvectors of C as its column, so that D ends up with eigenvalues along the diagonal.

4. Transform data to new basis:
Transform all the wire physics asymmetry (calculated over the entire data set) in the new frame using the matrix S :

$$\bar{B}_i = S_{j,i}\bar{A}_j \tag{26}$$

Where $S_{j,i}$ is the transpose of S.

5. Transform the formalism (fit) :
Now in the rotated (uncorrelated) frame, the the $y \to x$ map is not flat any more. Transforming everything we have :

Mean :

$$\bar{B}^{tot} = (K^T D^{-1} K)^{-1} K^T D^{-1} \bar{B} \tag{39}$$

Uncertainty :

$$(\Delta\bar{B}^{tot})^2 = (K^T D^{-1} K)^{-1} \tag{40}$$

Chi square :

$$\begin{aligned} \chi^2 &= (\bar{B} - \bar{B}^{tot})^T D^{-1} (\bar{B} - \bar{B}^{tot}) \\ &= \sum_i \frac{(\bar{B}_i - \bar{B}^{tot})^2}{\sigma_i^2} \end{aligned} \tag{41}$$

where,

$$W = D^{-1} = diag(\frac{1}{\sigma_1^2}, \frac{1}{\sigma_2^2}, \frac{1}{\sigma_3^2}, \frac{1}{\sigma_4^2}, ..., \frac{1}{\sigma_{144}^2}) \tag{42}$$

and D is the covariance matrix.

$$\bar{B} = S^T \bar{A} \tag{43}$$

$$K = S^T X \tag{44}$$

Since $\bar{A}_i$ or $\bar{B}_i$ is itself a mean of n pulse pairs , so D^{-1} is in fact $D^{-1} = diag(\frac{n}{\sigma_1^2}, \frac{n}{\sigma_2^2}, \frac{n}{\sigma_3^2}, \frac{n}{\sigma_4^2}, ..., \frac{n}{\sigma_{144}^2})$ in our analysis.

6. Quote the result :
So we quote the final result as :

$$B^{phy} = \bar{B}^{tot} \pm \Delta \bar{B}^{tot} \tag{45}$$

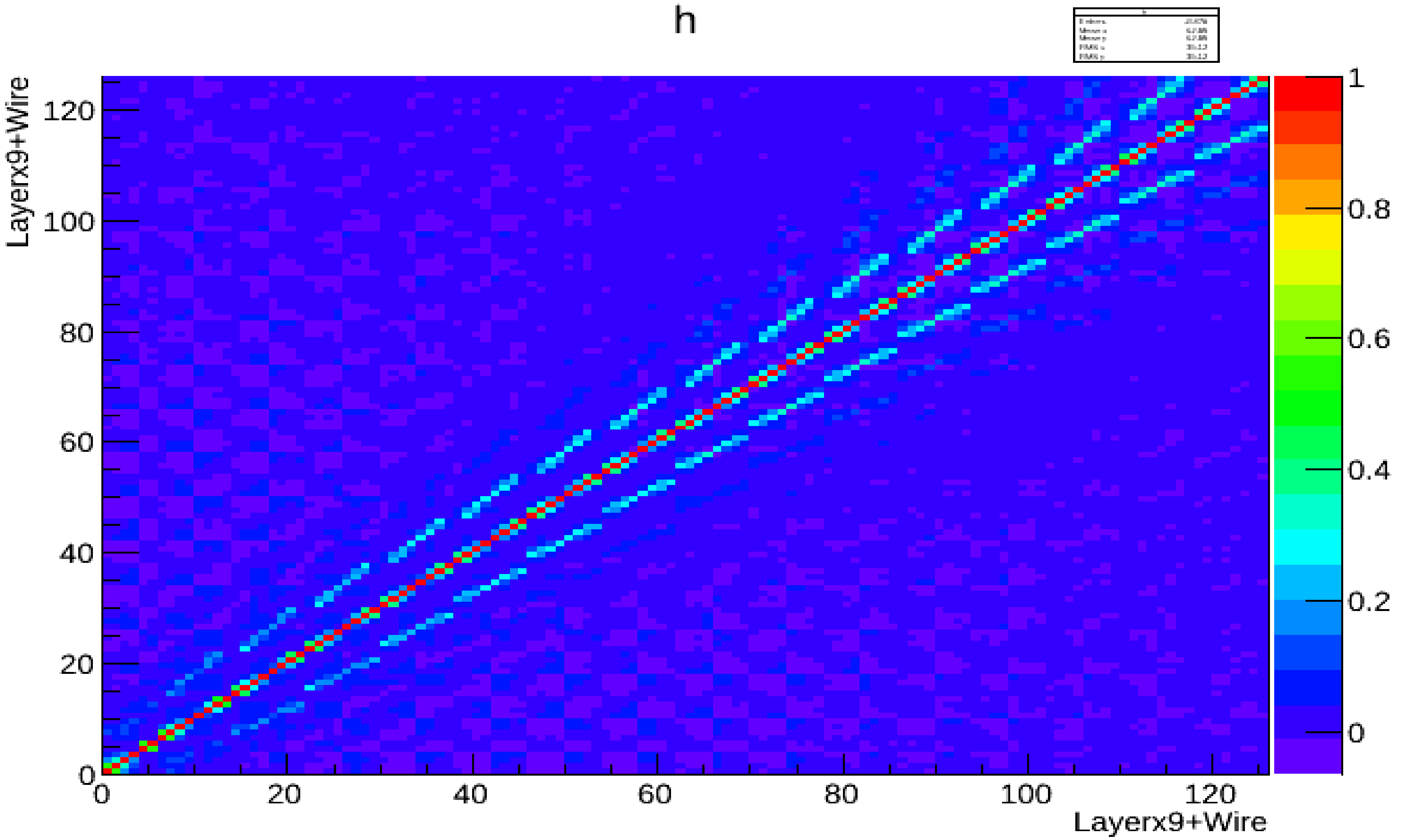
Origin of The Problem that failed previous attempts

- Our Previous attempts failed because of the following reason ---
  – The matrix elements that correspond to center wires or bad wires made the matrix nearly singular.
  – Inverting or diagonalizing a matrix with few elements very very small gave rise to unexpected outcome.

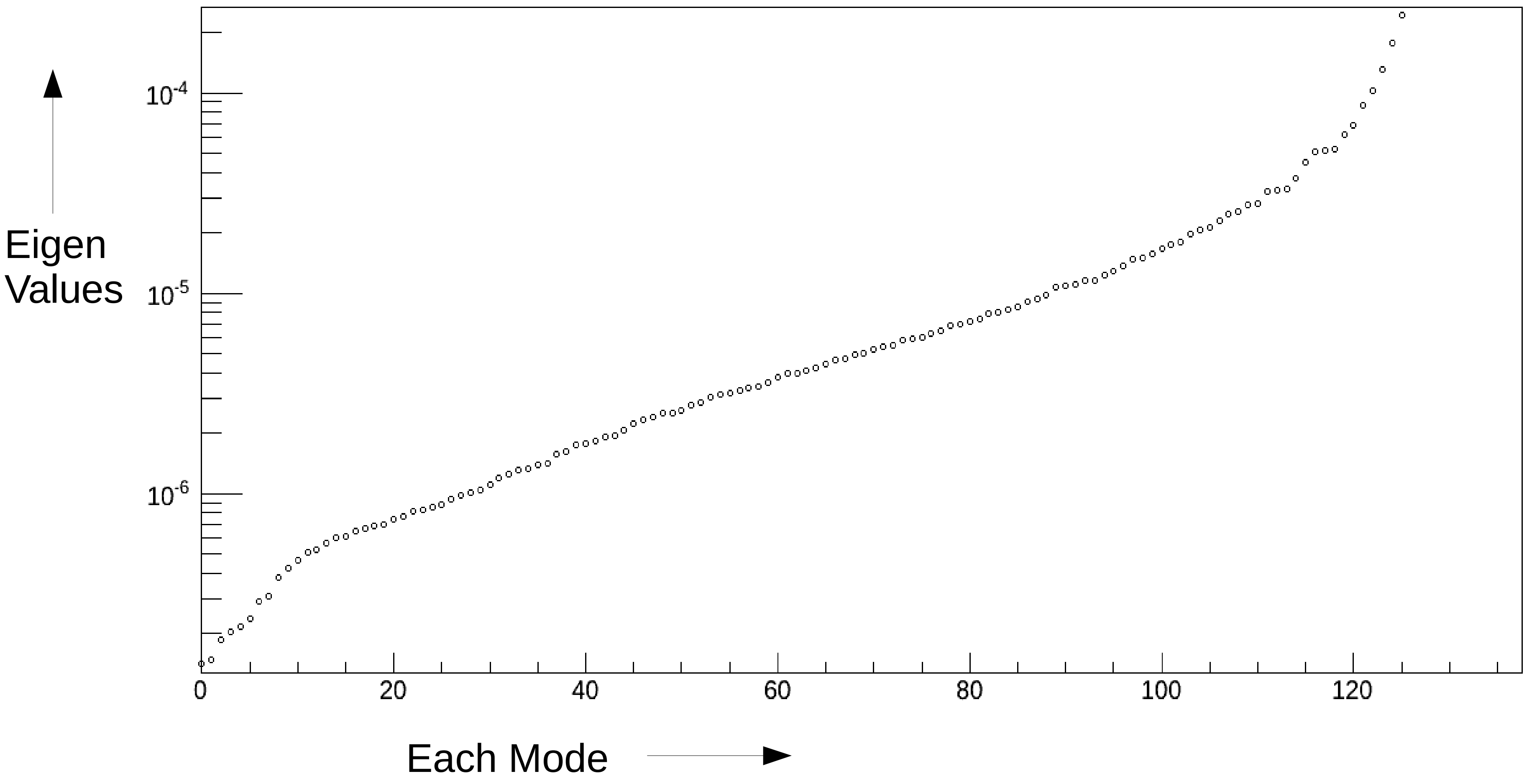
- The result seems to make sense if we just work with the remaining 126 wires (with 16 center and 2 bad wires excluded).

- Here the result is presented with “sum over detectors” normalized Physics asymmetry following the approach suggested by David.

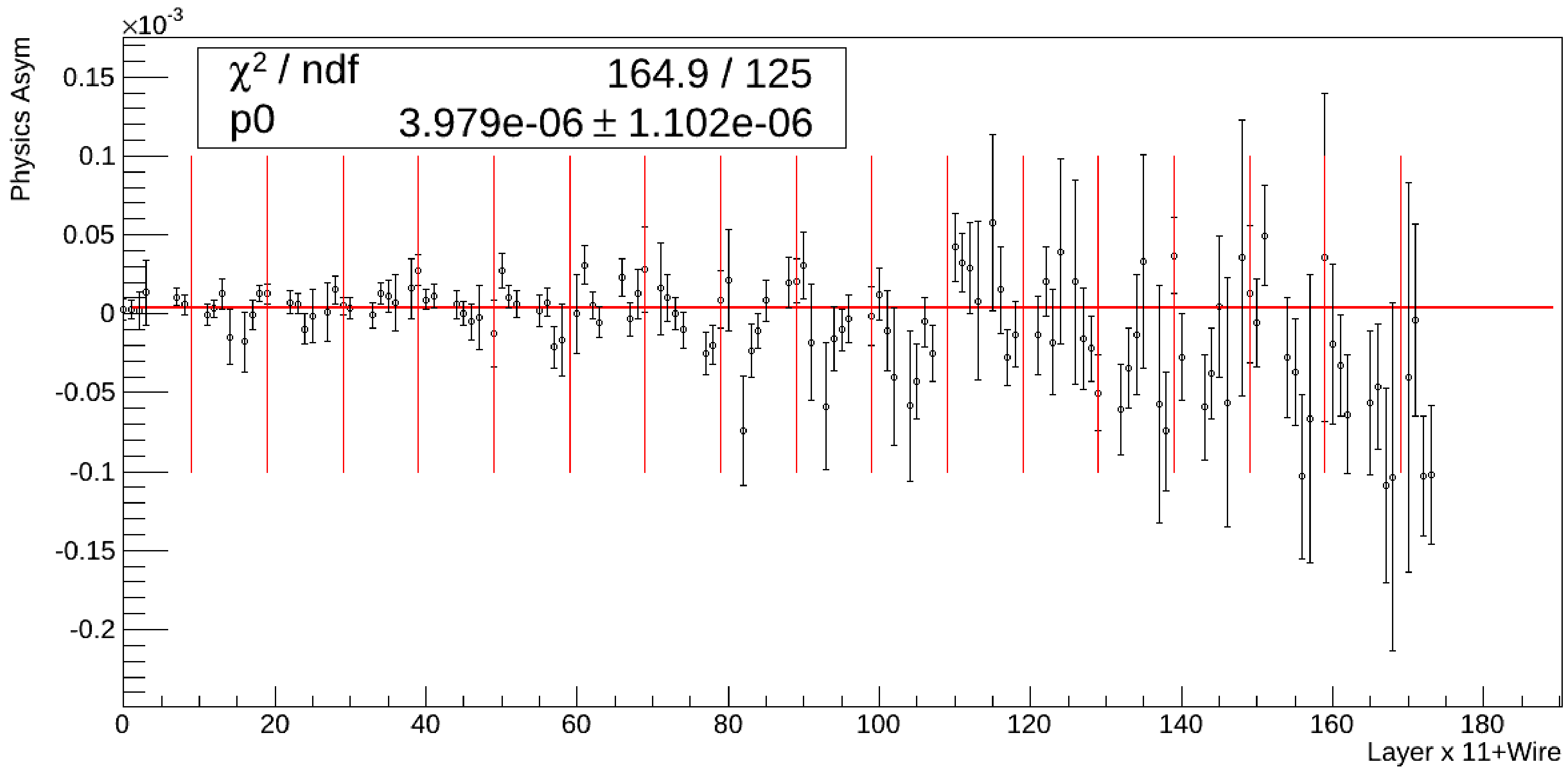
Physics Asymmetry Correlation



Eigenvalues from diagonalization



Correlated Physics Asymmetry (Run#26230)



Correlated vs Corrected (Run : 26230)

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Fit from correlated signals :

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Minimizer is Linear

Chi2 = 164.884

NDf = 125

p0 = 3.97886e-06 +/- 1.10173e-06

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Fit from Uncorrelated (Corrected) signals :

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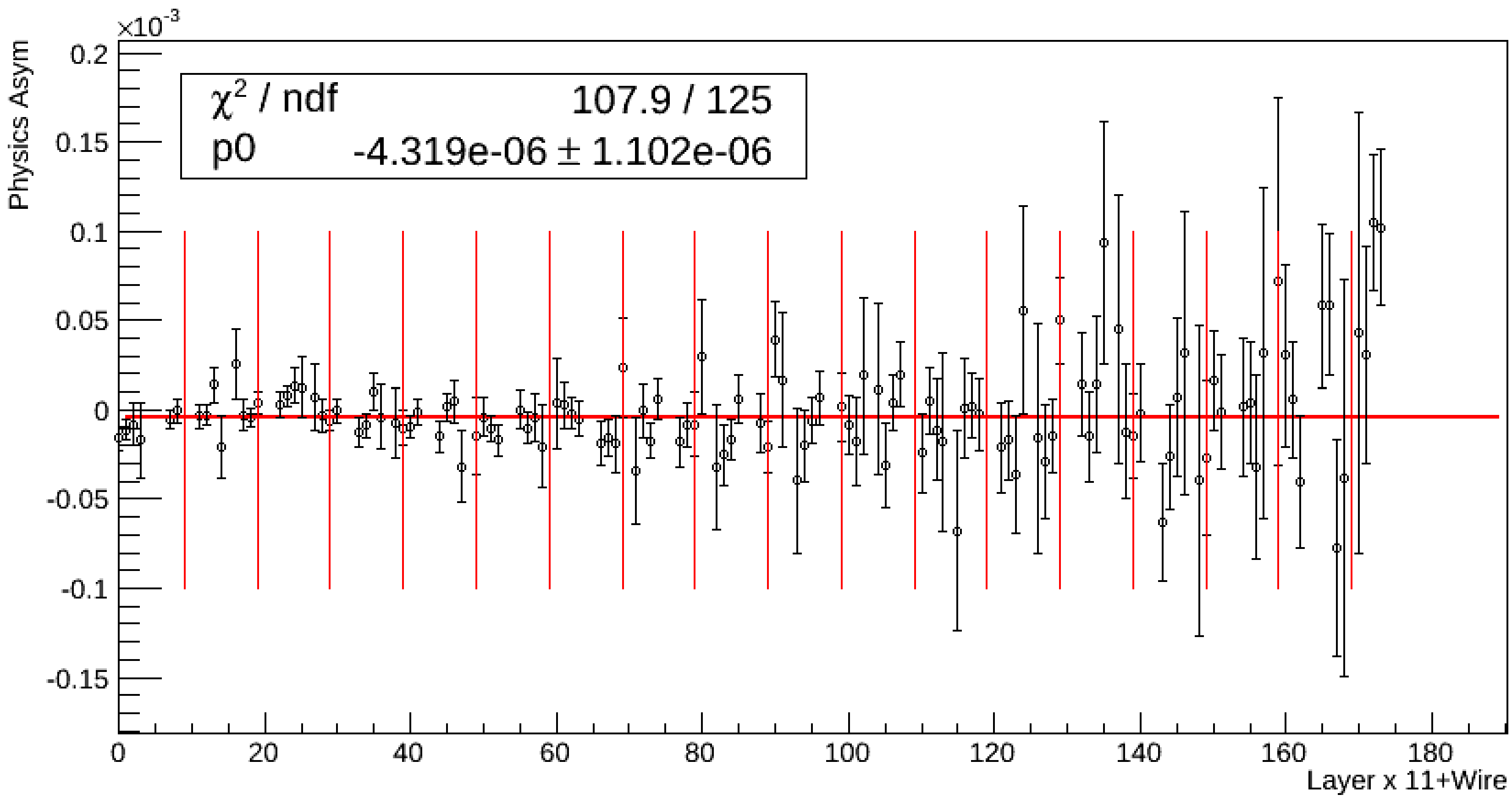
Mean : 4.64902e-06

Uncertainty : 1.74034e-06

Chi Square : 182.686

Correlated Physics (run#26231)



Correlated vs Corrected (Run : 26231)

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Fit from correlated signals :

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Minimizer is Linear
Chi2 = 107.944
NDf = 125
p0 = -4.31872e-06 +/- 1.10217e-06

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Fit from Uncorrelated signals :

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Mean : -4.96752e-06
Uncertainty : 1.70662e-06
Chi Square : 163.429