Correlations Calculation

The Recipe : First Approach

1. Calculate covariance of yields :

The covariance between the yields by two wires is given by -

$$Cov(Y^i, Y^j) = \frac{1}{N}\sum_{k=1}^{N}(Y_k^i - \bar{Y}^i)(Y_k^j - \bar{Y}^j) \tag{11}$$

Where i and j are wire the number indices and N is the number of samples. So $Cov(Y^i, Y^j)$ is a 144x144 matrix.

2. Calculate covariance of raw and physics asymmetry:

Since raw asymmetry A_i^r is related to the yield by the relation,

$$A_i^r = \frac{Y_i^{\uparrow} - Y_i^{\downarrow}}{Y_i^{\uparrow} + Y_i^{\downarrow}} \tag{12}$$

So we can relate the covariance in yields to the raw asymmetry as

$$Cov(A_i^r, A_j^r) = \frac{Cov(Y_i, Y_j)}{\bar{Y}_i \bar{Y}_j} \tag{13}$$

Any the physics covariance is

$$Cov(A_i^p, A_j^p) = \frac{Cov(A_i^r, A_j^r)}{G_i G_j} \tag{14}$$

superscript "r" indicates raw asymmetry and "p" for physics asymmetry. Indices i and j again for wire number. G indicates the geometry factor.

3. Calculate total (overall) physics asymmetry:

Now the total (global) physics asymmetry is related to individual wire physics asymmetry by –

$$A_p^{tot} = \frac{\sum_i w_i A_i^p}{\sum_i w_i} \tag{15}$$

where the weight w_i is given by—

$$w_i = \sum_j InvCov(A_i^p, A_j^p) = \sum_j G_i G_j InvCov(A_i^r, A_j^r) = \sum_j G_i G_j \bar{Y}_i \bar{Y}_j InvCov(Y_i, Y_j) \tag{16}$$

Where InvCov is the inverse of the covariance matrix.

4. Calculate error in total(overall) physics asymmetry :

The error in the total(overall) physics asymmetry –

$$\Delta A_p^{tot} = \frac{1}{\sqrt{\sum_i w_i}} \tag{17}$$

where w_i is given by equation (16).

5. Calculate the chi-square of the weighted average :
The chi-square in our weighted asymmetry calculation is given by –

$$\chi^2 = \sum_{ij} (A_i^p - A_p^{tot}) w_{ij} (A_j^p - A_p^{tot}) \quad (18)$$

6. Quote the Result:
Then the final result is :

$$A_p^{tot} \pm \Delta A_p^{tot} \quad (19)$$

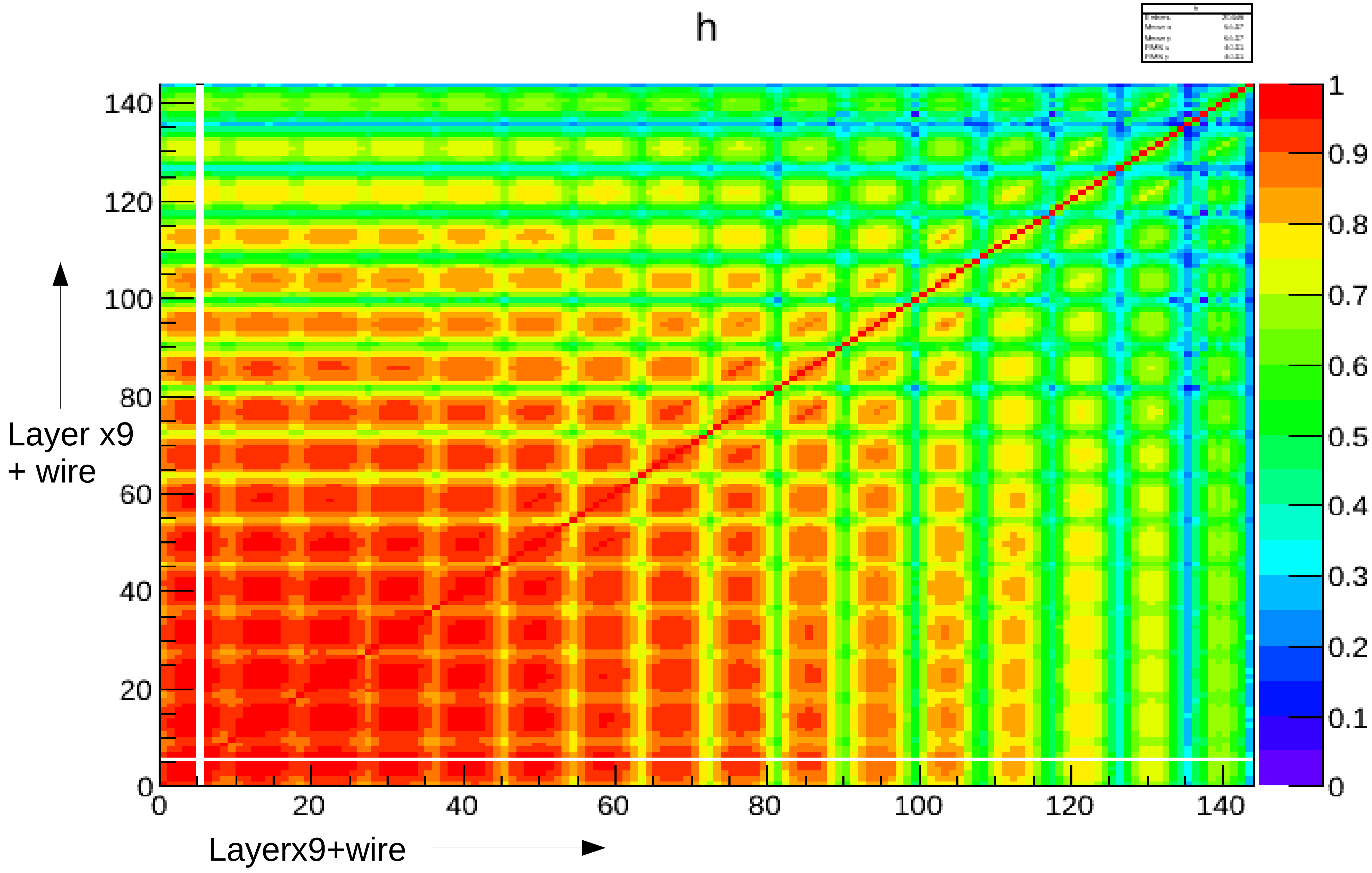
And the goodness of the result is given by χ^2/ndf

Now the measure of covariance is often expressed as the correlation coefficient (which is just the normalized covariance) as :

$$\rho = \frac{Cov(Y_i, Y_j)}{\sigma_i \sigma_j} \quad (20)$$

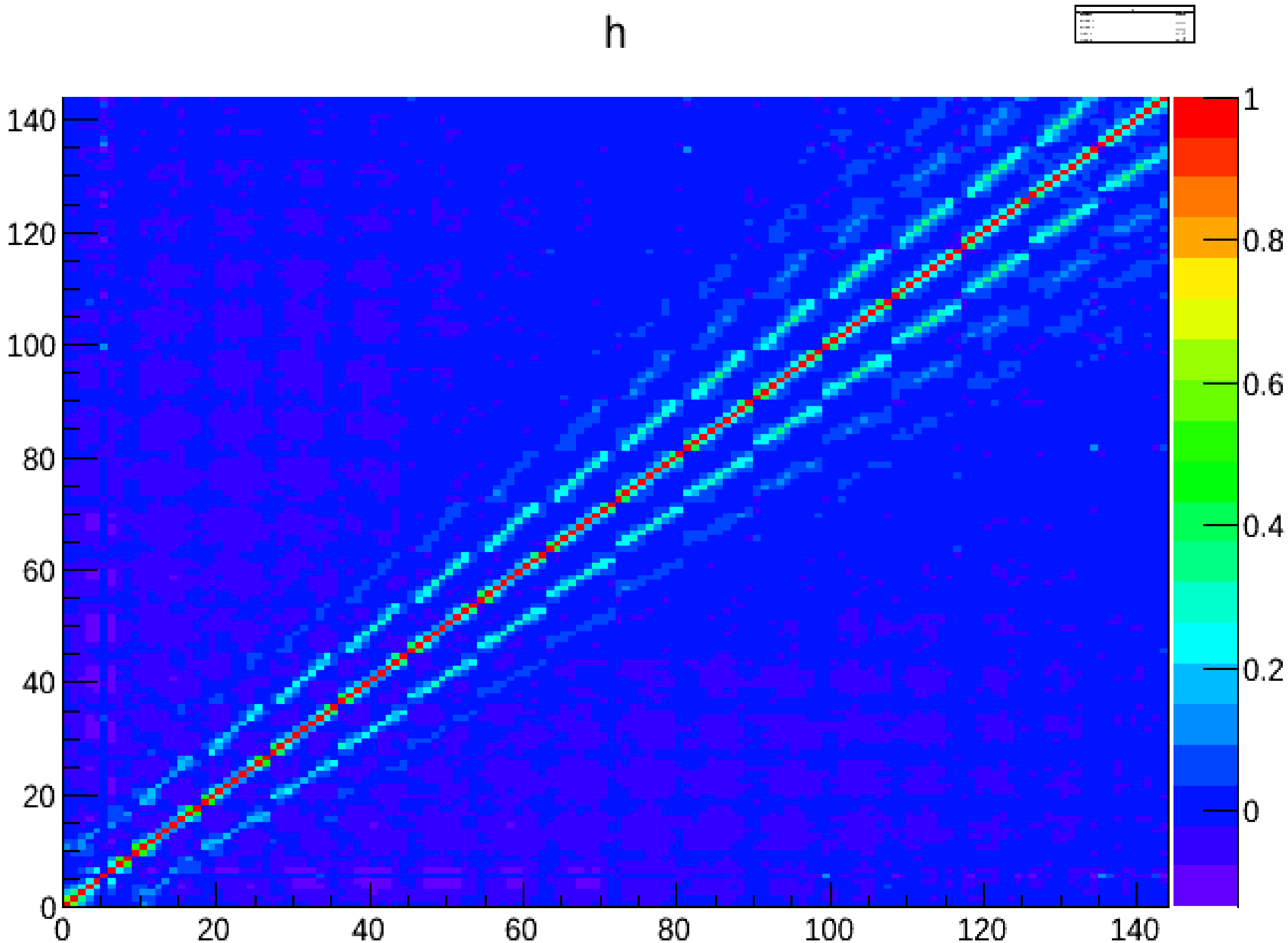
where σ_i and σ_j are standard deviations (square root of variance). The correlation coefficient varies between -1 and +1 where the sign indicates the sense of correlation. If the variables are perfectly correlated linearly , then $|\rho| = 1$, if the variables are independent then $\rho = 0$(care should be taken

Yield Correlations (**Without sum over detectors normalization**): Typical UD run: run# 26230 at 0.5 atm

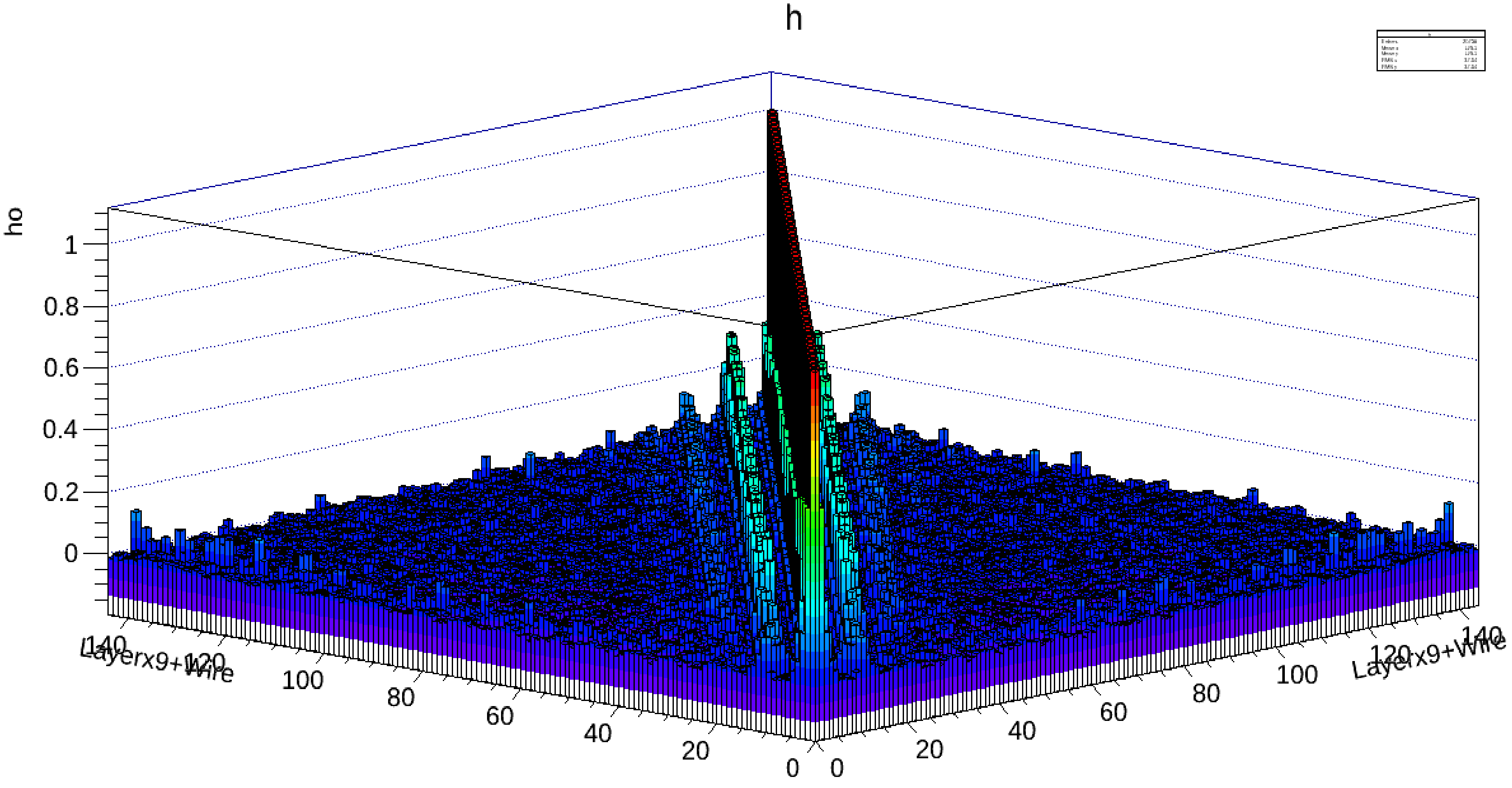


Yield Correlations (**with yields normalized by sum over detectors**):Typical UD run: run# 26230 at 0.5 atm

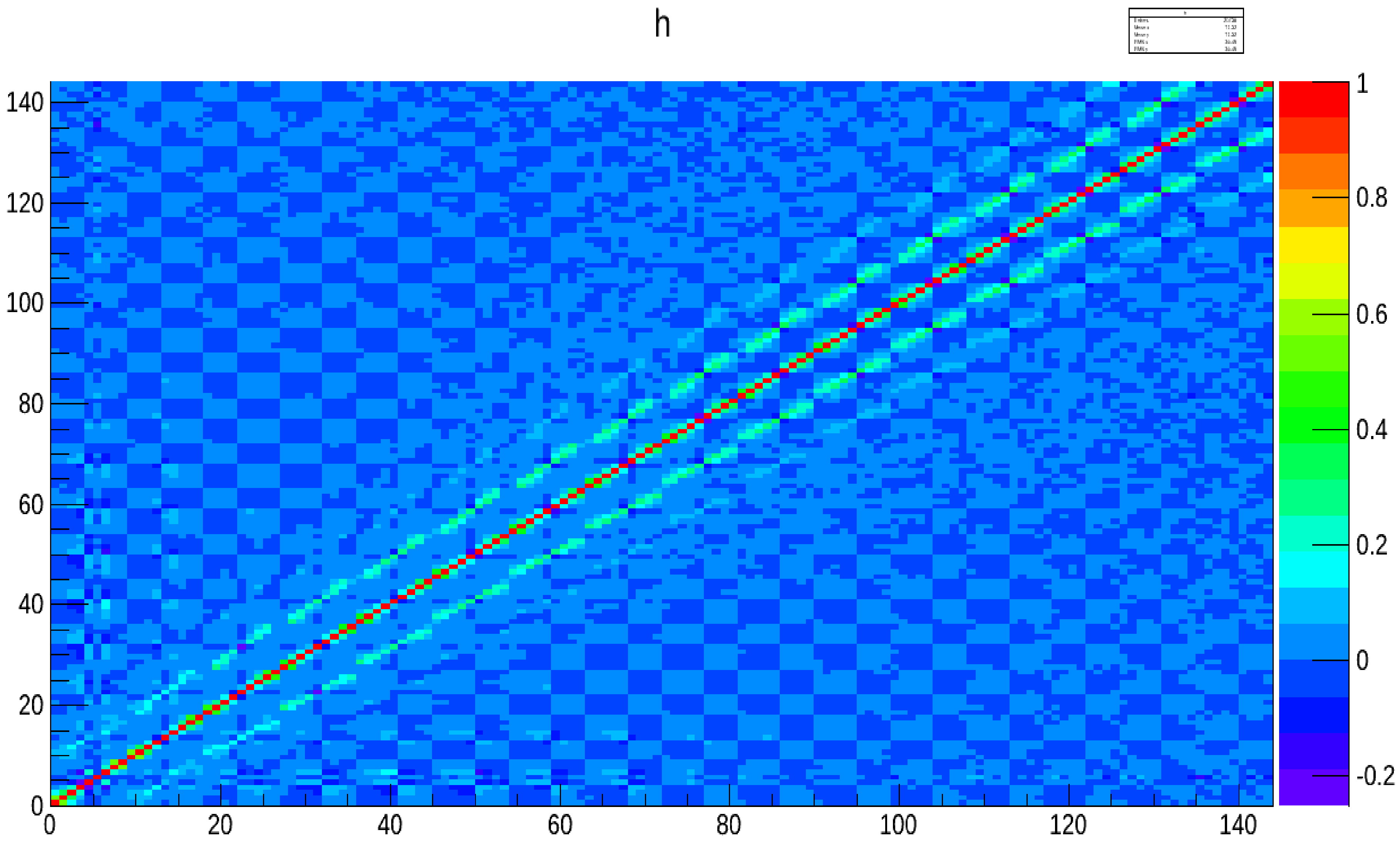
(Alternative plotting)



Yield Correlations :UD run 0.5 atm : run#26230
(Alternative Plotting)



Physics Asymmetry Correlations (First Approach)



Correlation Calculation : Through Diagonalization

The Recipe :

1. Calculate the physics asymmetry :

For a run, calculate the physics asymmetry between two pulses of any wire. The yield in the asymmetry has to be sum over detectors normalized and pedestal subtracted.

$$A_i = \frac{1}{G_i}\frac{Y_i^{\uparrow} - Y_i^{\downarrow}}{Y_i^{\uparrow} + Y_i^{\downarrow}} \tag{22}$$

2. Calculate the covariance out of physics asymmetry :

$$C_{ij} = Cov(A^i, A^j) = \frac{1}{N}\sum_{k=1}^{N}(A_k^i - \bar{A}^i)(A_k^j - \bar{A}^j) \tag{23}$$

So C_{ij} is a 144x144 matrix and by construction its a symmetric matrix.

3. Diagonalize the covariance matrix :

Noting the fact that matrix C is symmetric, find out a matrix S such that –

$$S^T C S = D \tag{24}$$

Where D is a diagonal matrix wit diagonal elements

$$D = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2 ..., \sigma_{144}^2) \tag{25}$$

The matrix S is the transformation matrix to be used to transform back and forth between the correlated and uncorrelated basis(reference frame).

The matrix S can be built up from the eigenvectors of C as its column, so that D ends up with eigenvalues along the diagonal.

4. Transform data to new basis:

Transform all the wire physics asymmetry (calculated over the entire data set) in the new frame using the matrix S :

$$\bar{B}_i = S_{i,j}\bar{A}_j \tag{26}$$

5. Calculate total (overall) asymmetry :

Now the total(overall) asymmetry and its uncertainty can be calculated using formulas for uncorrelated data as :

$$\bar{B}^{tot} = \frac{\sum_i B_i \frac{1}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}} \tag{27}$$

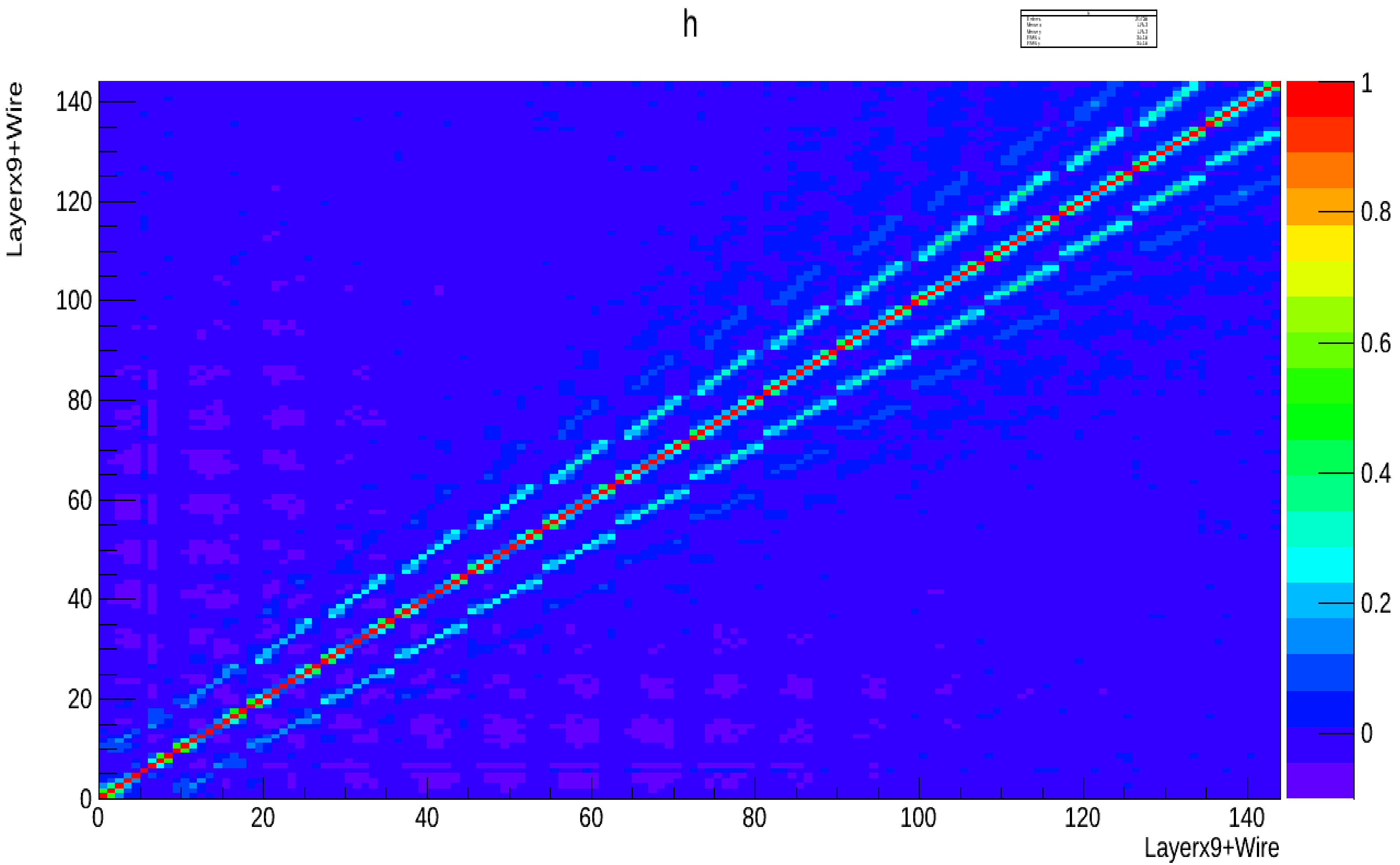
$$\Delta\bar{B}^{tot} = \frac{1}{\sqrt{\sum \frac{1}{\sigma_i^2}}} \tag{28}$$

Also calculate the chi square to be compared for goodness of fit :

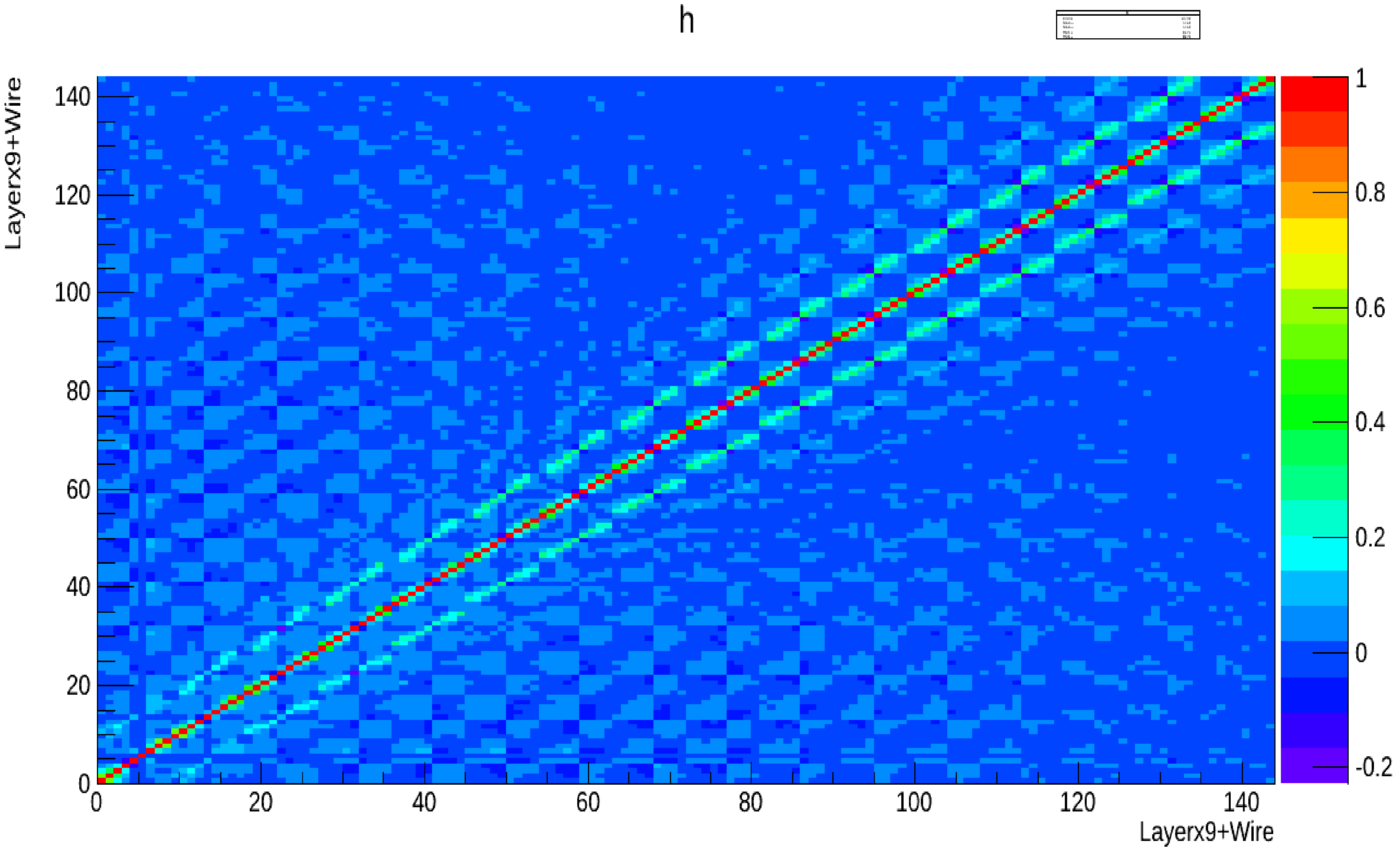
$$\chi^2 = \sum_i \frac{(B_i - \bar{B})^2}{\sigma_i^2} \tag{29}$$

Alternatively, instead of calculating the above three formulas, just make a plot of B_i's and then fit to y=mx and calculate the fit parameters.

Raw Asymmetry Correlations



Physics Asymmetry Correlations



- Diagonalization method gives physics asymmetry and it's error ~ 10^{-5} from one run.
- Open questions : How to include A and B set data with diagonalization method ?