Spin Flipper
and
Neutron Polarimetry
for the
n^3He Experiment

Christopher Hayes

April 6th 2016

Outline

1. Theoretical Background

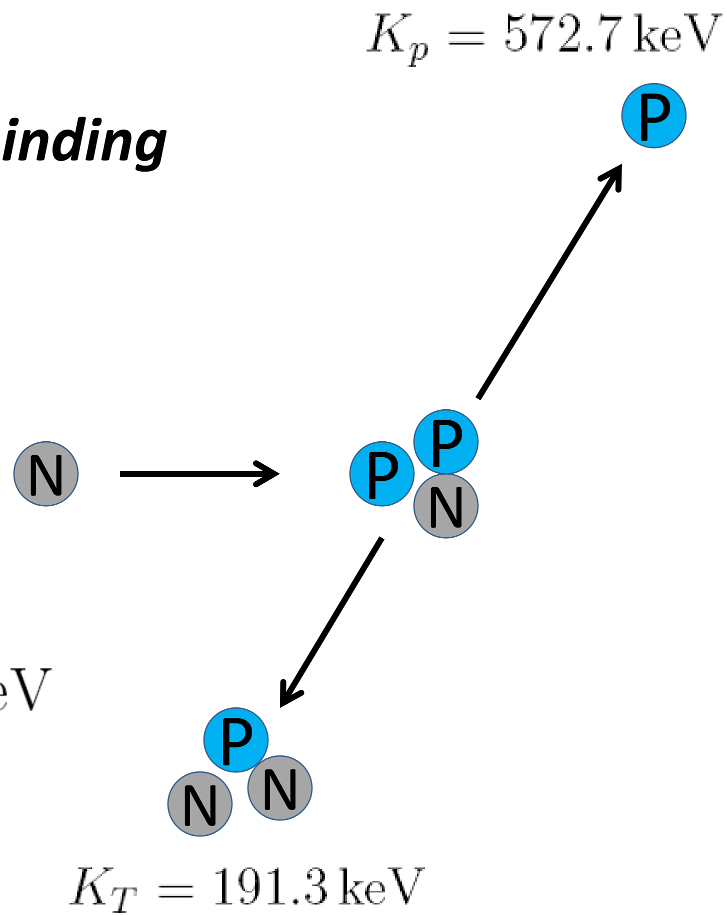
2. SNS neutrons/Experiment Overview

3. Spin Flipper

4. Neutron Polarimetry

Interaction of Neutrons with ^{3}He

$$\vec{n} + {}^3He \; = \; {}^3H + p + 764\,\mathrm{keV}$$



- ***Binding energy of He-3 is less than Binding energy of Triton***

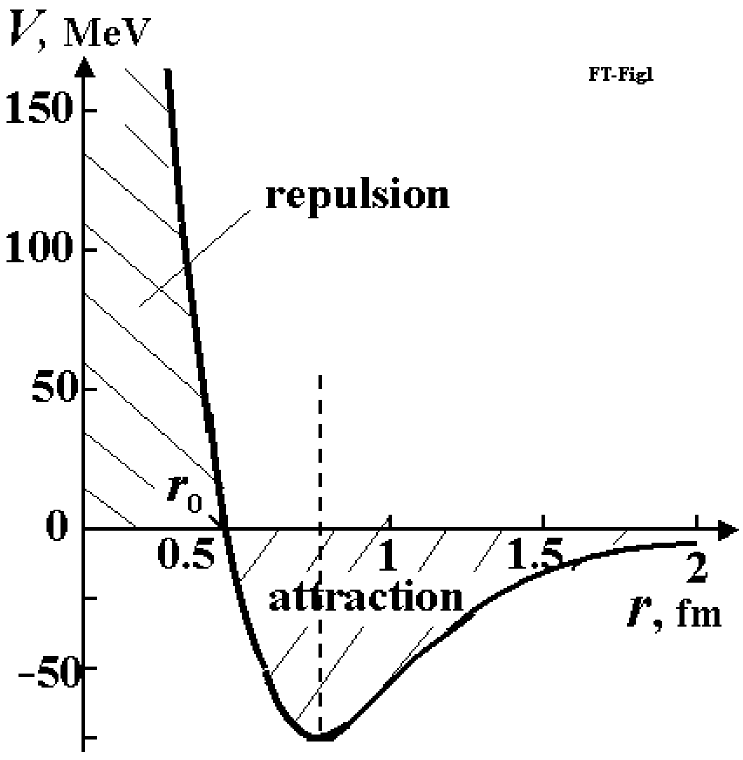
$$E_b[T] = (M_p + 2M_n - M_T)c^2$$

$$E_b[He3] = (M_n + 2M_p - M_{He3})c^2$$

$$K = (M_{He3} + M_n - M_T - M_p)c^2 = 764\,\mathrm{keV}$$

Theory of the Nuclear Force

- ***Also known as the residual Strong Interaction***
- ***Attractive force mediated by the exchange of light virtual mesons*** $\pi,\ \rho,\ \omega$
- ***Non-central force (Tensor couplings)***
- ***Spin-Dependent***

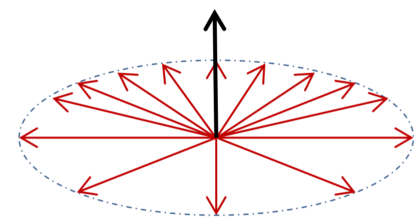


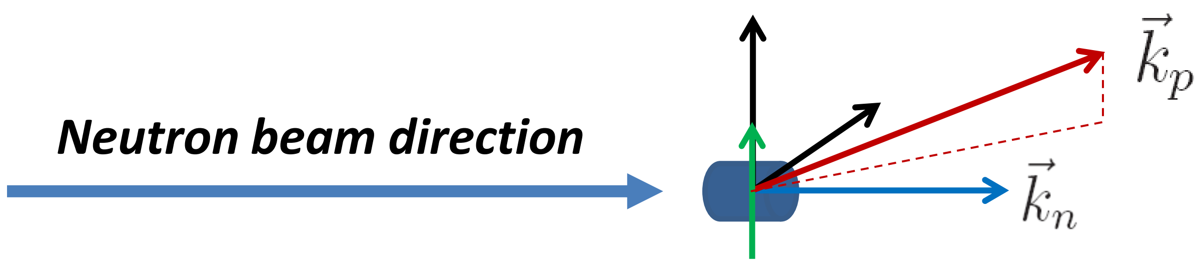
Differential Cross-Section

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}\left[1 + \varepsilon_{pc} + \varepsilon_{pv} + \cdots\right]$$

$$\varepsilon_{pc} = \alpha_{pc}\langle\boldsymbol{\sigma}\rangle \cdot (\boldsymbol{k}_n \times \boldsymbol{k}_p)$$

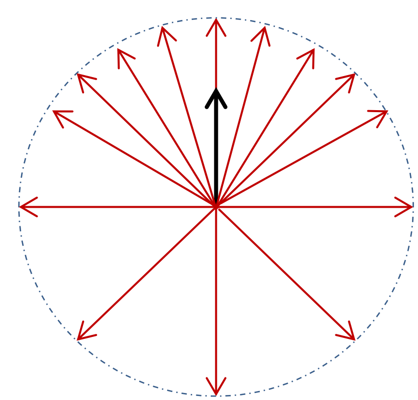
L/R asymmetry





$$\varepsilon_{pv} = \alpha_{pv}\langle\boldsymbol{\sigma}\rangle \cdot \boldsymbol{k}_p$$

U/D asymmetry

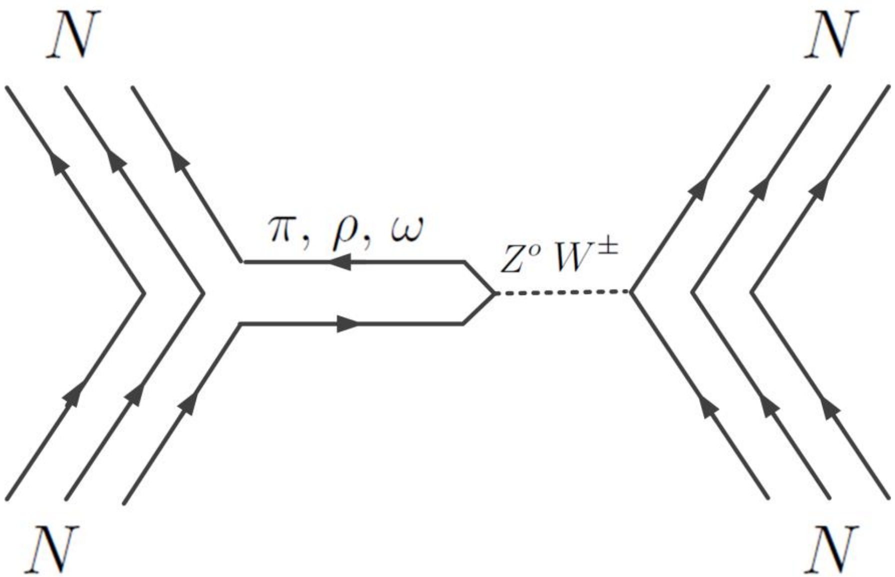


Theory of the Hadronic Weak Interaction

- ***HWI described by the Meson Exchange Model.***

- ***Mediators*** $\pi,\ \rho,\ \omega$ ***allowed to decay into carriers of the weak force before coupling to second vertex***

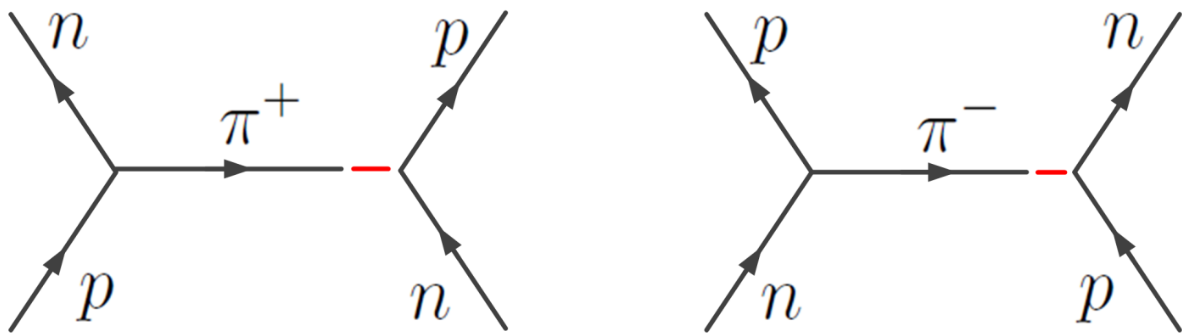
- ***Theory requires 6 coupling constants which must be measured experimentally.***



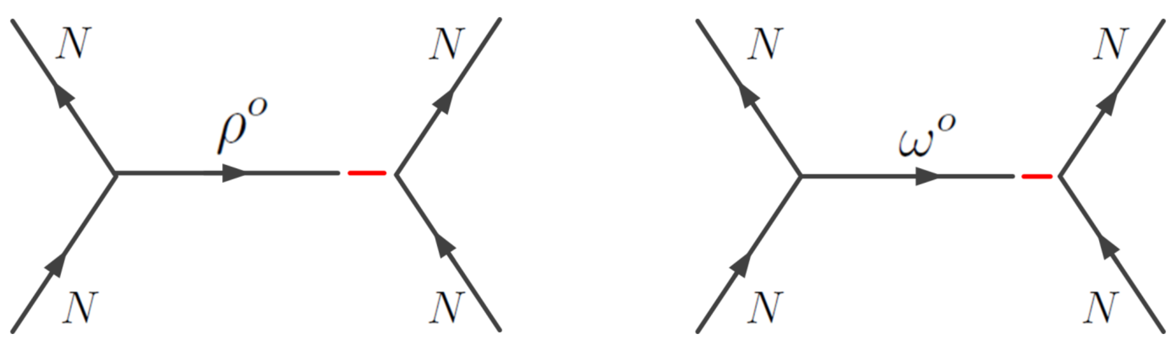
$$A = a_\pi^1 \cdot h_\pi^1 + a_\rho^0 \cdot h_\rho^0 + a_\rho^1 \cdot h_\rho^1 + a_\rho^2 \cdot h_\rho^2 + a_\omega^0 \cdot h_\omega^0 + a_\omega^1 \cdot h_\omega^1$$

Feynman Diagrams which determine A_p

$$\boxed{A_p = -0.18 \cdot h_\pi^1 - 0.14 \cdot h_\rho^0 - 0.13 \cdot h_\omega^0}$$



HWI mediated by charged pion exchange



HWI mediated by the exchange of heavy neutral vector mesons

Observables and Coupling Coefficients

- ***n^3He is one of several experiments designed to measure coupling constants of the DDH model***
- ***Primary goal of NPDGamma experiment is to measure the parity violating Gamma asymmetry:***

$$\vec{n} + p \to d + \gamma$$

$$A_\gamma \approxeq -0.107 h_\pi^1$$

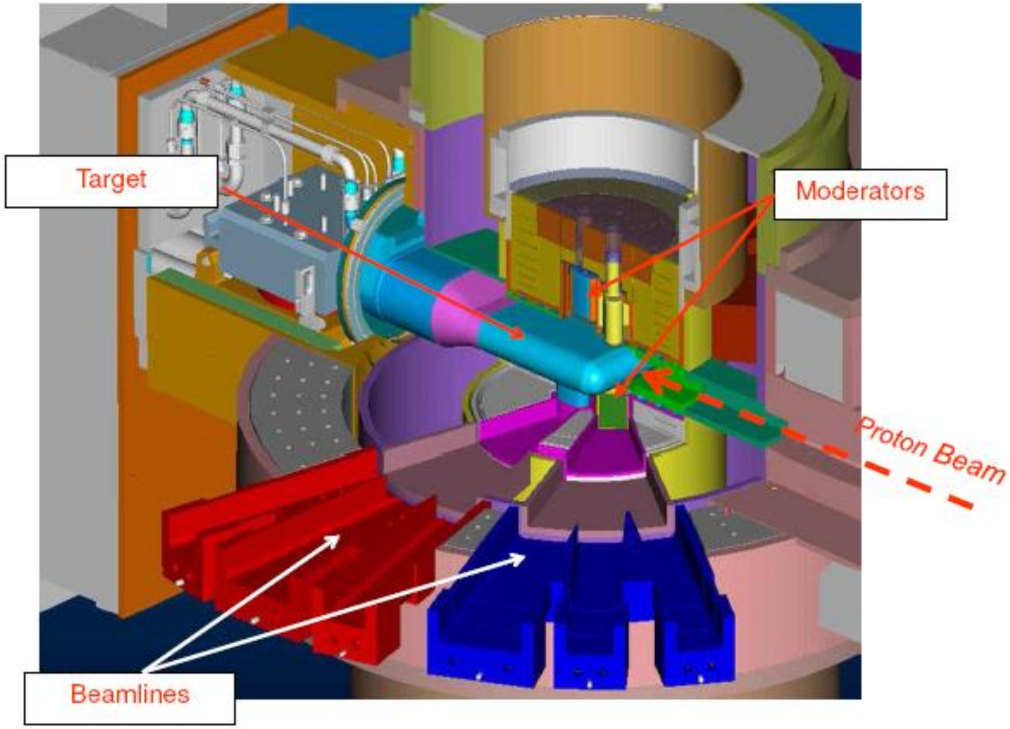
	np A_γ	nD A_γ	n^3He A_p	np ϕ	nα ϕ	pp A_z	pα A_z
f_π	-0.11	0.92	-0.18	-3.12	-0.97		-0.34
h_ρ^0		-0.50	-0.14	-0. 23	-0.32	0.08	0.14
h_ρ^1	-0.001	0.10	0.027		0.11	0.08	0.05
h_ρ^2		0.05	0.0012	-0.25		0.03	
h_ω^0		-0.16	-0.13	-0. 23	-0.22	-0.07	0.06
h_ω^1	-0.003	-0.002	0.05		0.22	0.07	0.06

- ***HWI also characterized by EFT couplings known as Danilov Parameters*** $\lambda_t,\ \lambda_s^0,\ \lambda_s^1,\ \lambda_s^2,\ \rho_t,\ C_6^\pi$

SNS Neutrons/ Overview of Experiment

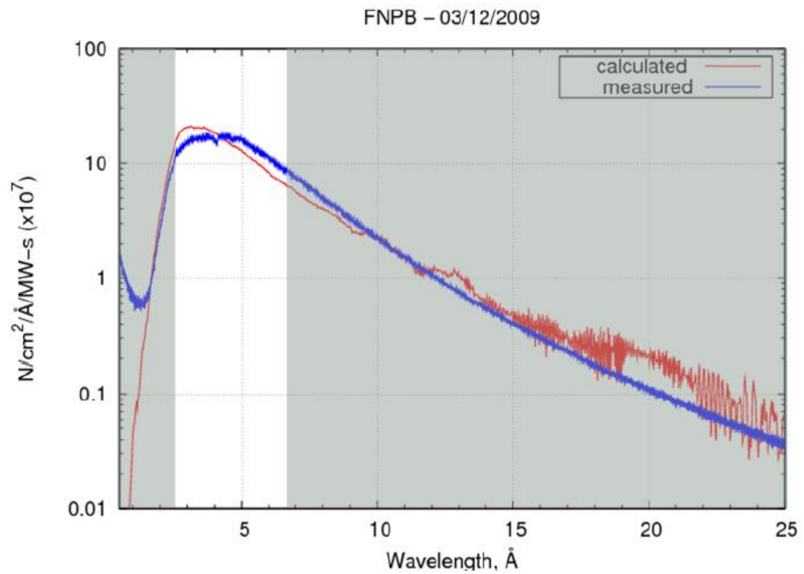
SNS Neutrons for FnPB

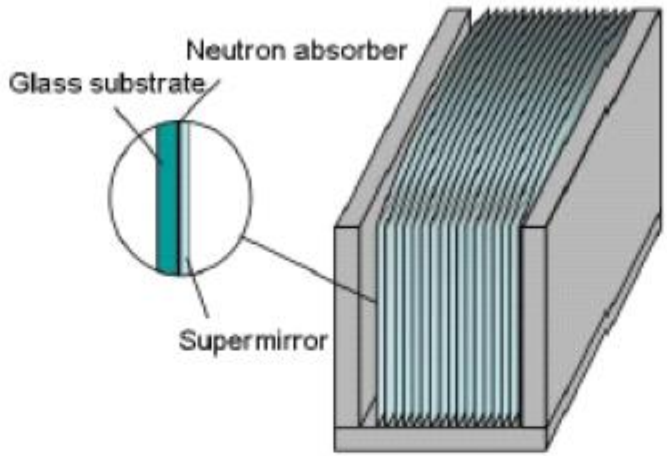
- ***Neutrons from a 60 Hz 1GeV proton Beam***
- ***1mA of current on target corresponds to a power of 1MW.***
- ***Cryogenic moderator at 20K produces cold neutrons***



SNS Neutrons for FnPB (II)

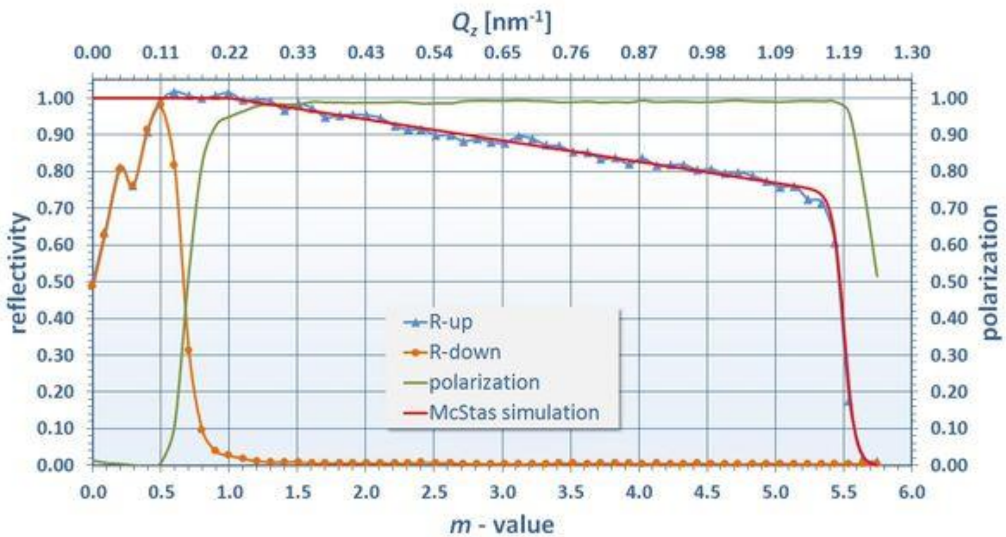
- ***Cold neutrons enter a Supermirror neutron guide***
- ***Gaps for frame definition choppers which transmit wavelengths 2.5 – 6.5A.***
- ***Supermirror Polarizer at end of guide***
- ***Polarizer reflects one neutron spin state and absorbs the other***

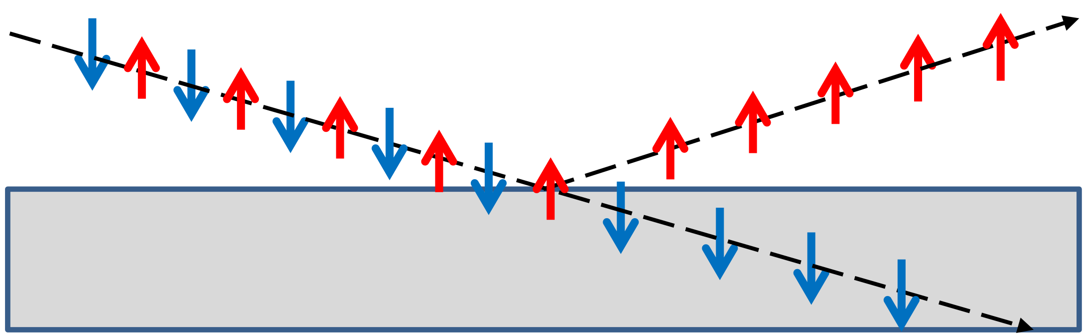




Supermirror Bender Polarizer

- ***Channels have radius of curvature 14.8 m***
- ***Every neutron incident on polarized will bounce at least once.***
- ***Supermirror coating composed of alternating layers of Ni and Si***
- ***Large external B-field saturates magnetization of Ni layers***





$$n = \sqrt{1 - \left(\frac{N b_{coh}}{\pi} \pm \frac{\mu B}{2\pi^2 \hbar^2} \right) \lambda^2}$$

Neutron Beam as Mixed Ensemble

- ***Beam neutrons emerging from the moderator have randomly oriented spins: Incoherent Mixture***
- ***Describe beam with a Density Matrix:***

$$\rho = \frac{1}{2}|S_z+\rangle\langle S_z+| + \frac{1}{2}|S_z-\rangle\langle S_z-|$$

- ***Expectation values for the random ensemble calculated using the density operator***

$$\langle S_x \rangle = Tr[\rho S_x] = 0$$

$$\langle S_y \rangle = Tr[\rho S_y] = 0$$

$$\langle S_z \rangle = Tr[\rho S_z] = 0$$

Neutron Beam after the SMP

- ***State vector neutron beam reflected from a perfect supermirror:***

$$|\psi\rangle_{beam} = |S_y+\rangle$$

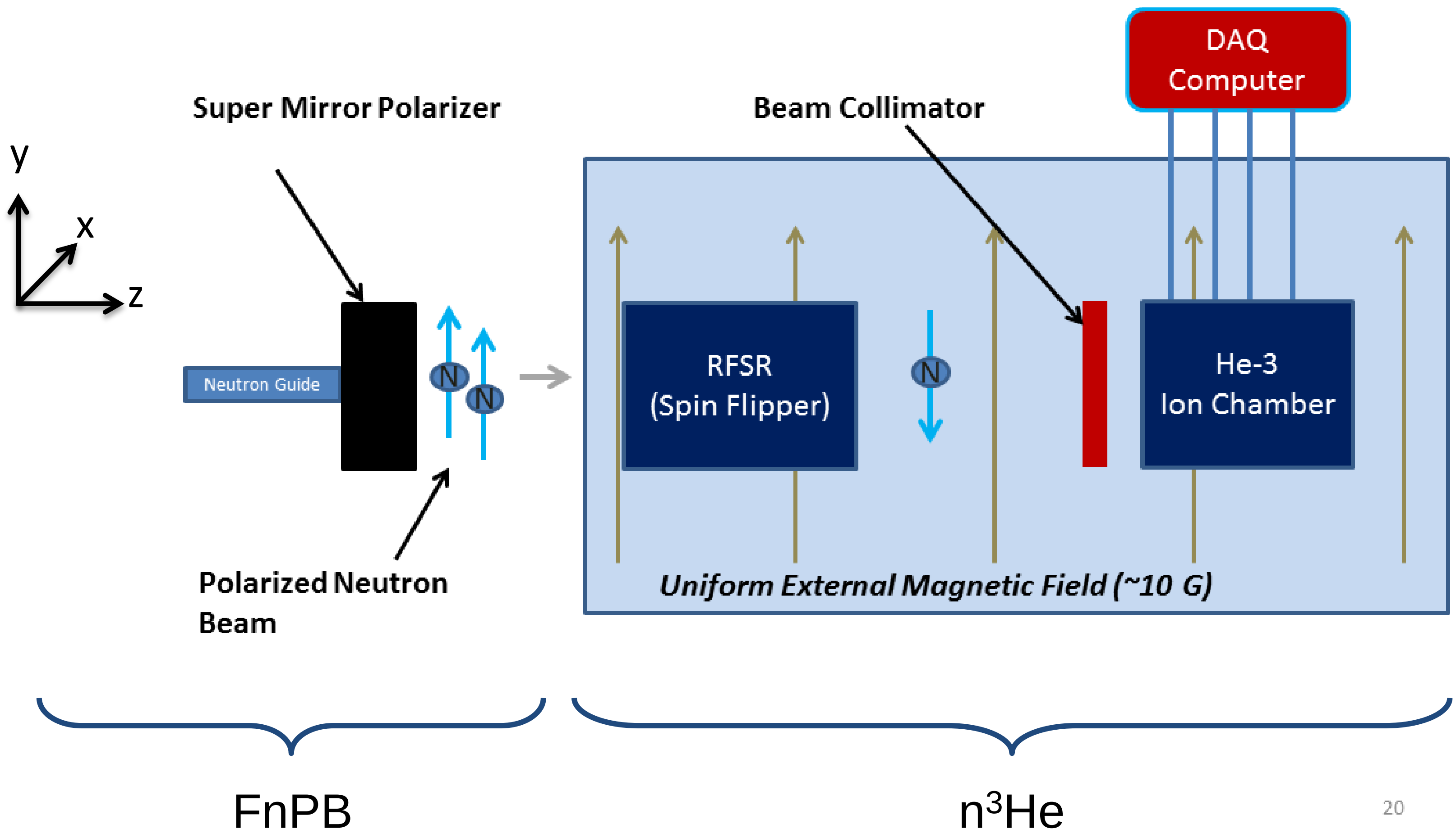
- fp ***Density operator for the beam at polarization P_n***

$$\rho = P_n|S_y+\rangle\langle S_y+| \; + \; \frac{1-P_n}{2}|S_x+\rangle\langle S_x+| \; + \; \frac{1-P_n}{2}|S_x-\rangle\langle S_x-|$$

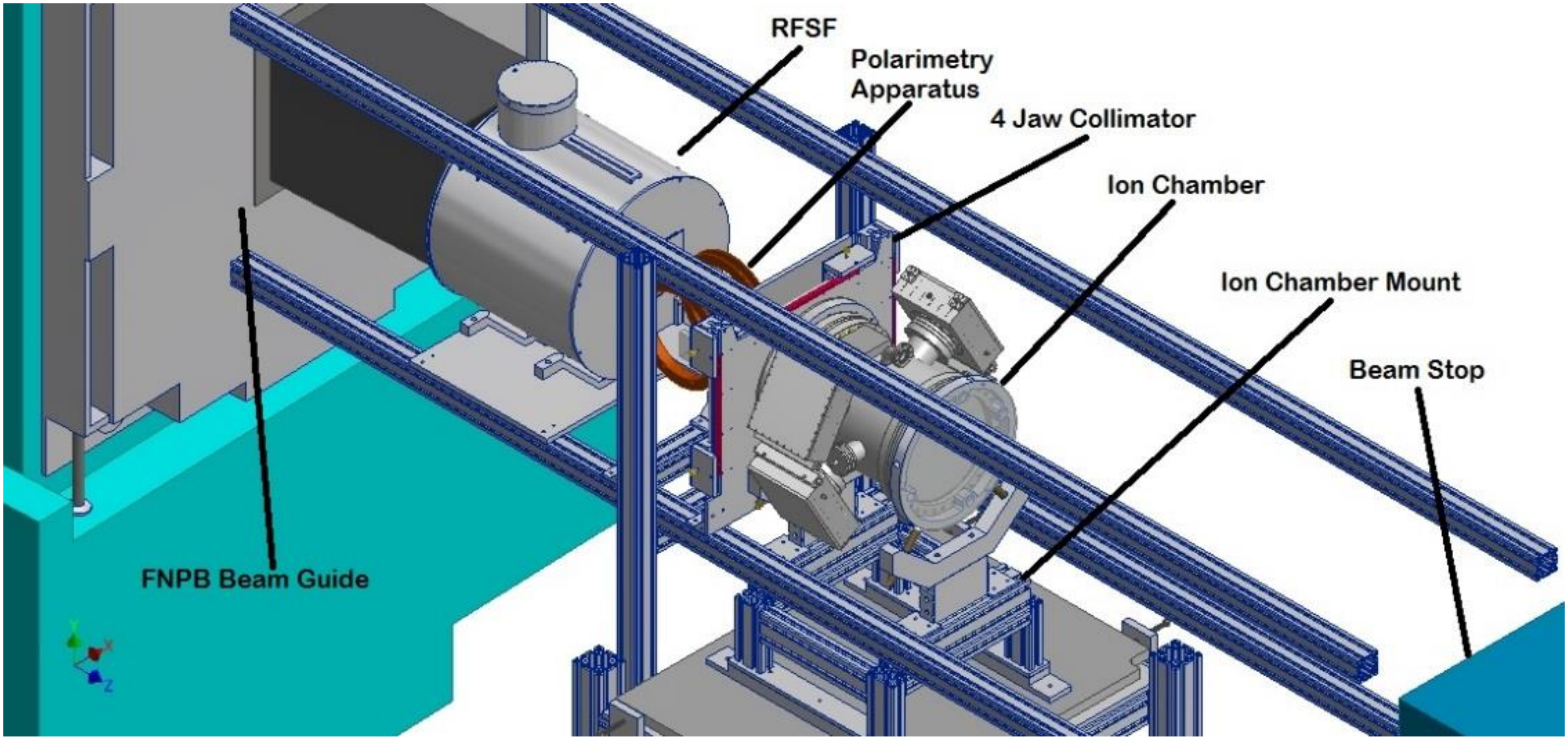
$$\langle S_x\rangle = Tr[\rho S_x] = 0$$
$$\langle S_z\rangle = Tr[\rho S_z] = 0$$

$$\langle S_y\rangle = Tr[\rho S_y] = \frac{\hbar}{2}P_n$$

Details of Experiment



Details of Experiment (II)



Ion Chamber

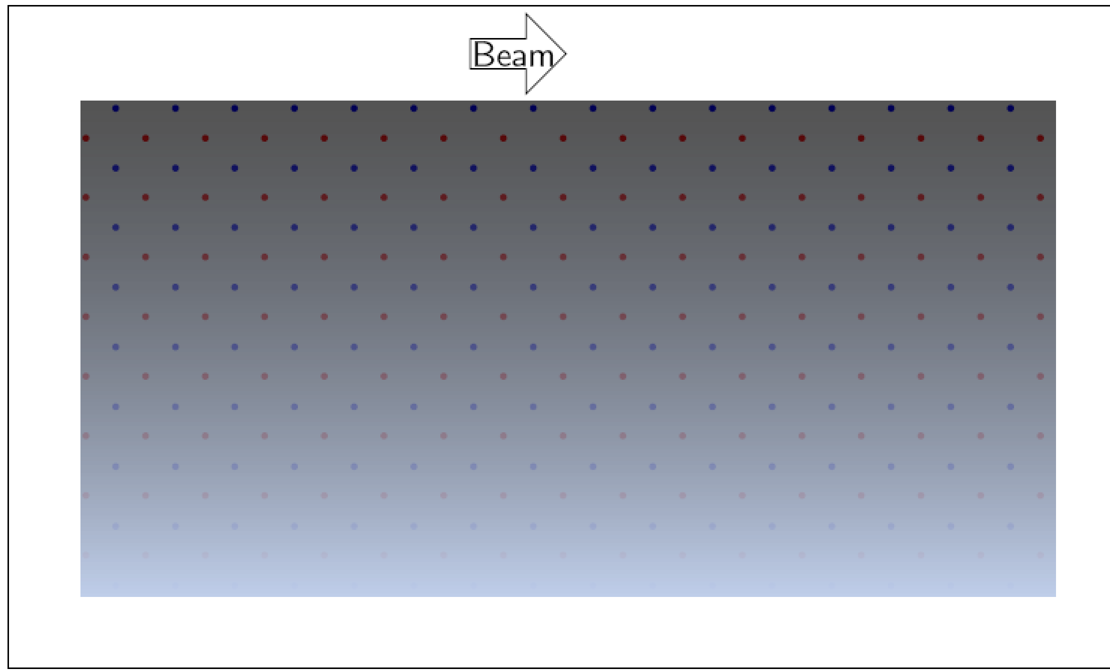
Frame stack holds alternating planes of high voltage and signal wires

- ***17 High Voltage Wire Planes***
- ***16 Signal Wire Planes***
- ***144 Signal Wires Total***

Wires in the ion chamber

- *Red wires are high voltage*
- *Blue wires are signal wires*

$$\langle \sigma \rangle$$



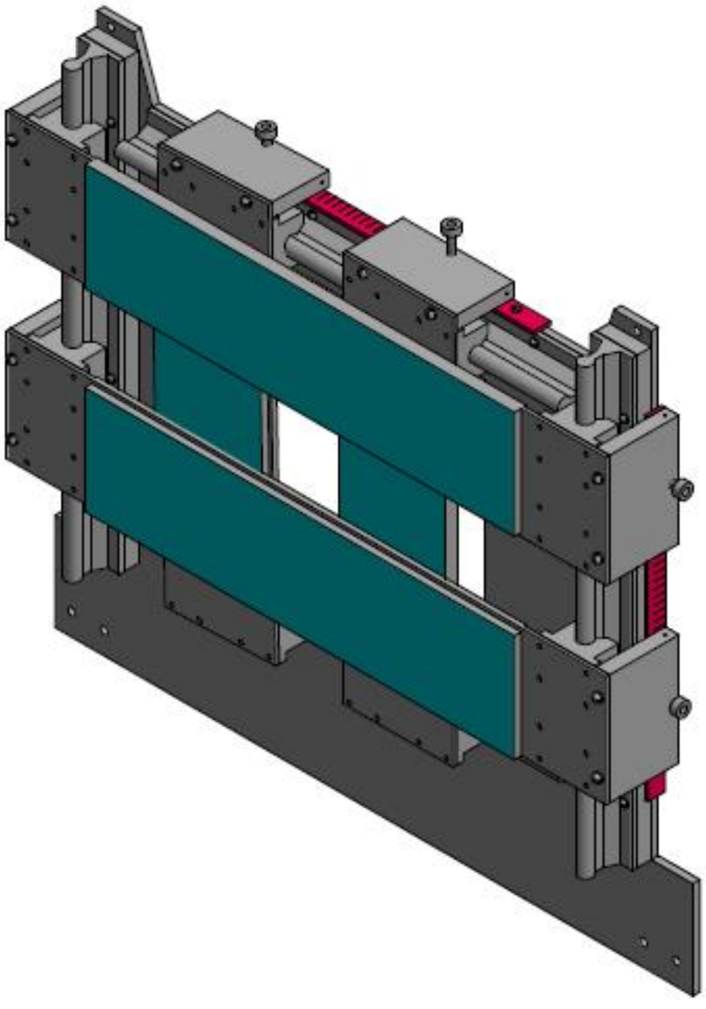
- HV 17 HV Frames with 8 wires each
- Signal 16 signal Frames with 9 wires each

Wires labeled by: (S,w)

- ***Spacing between all wires of the same color is 1.9cm***

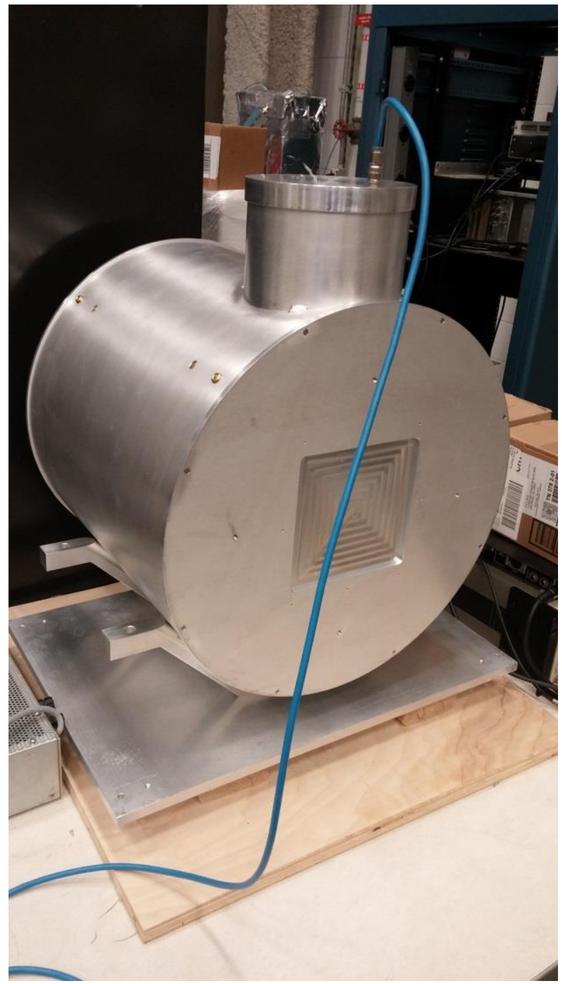
Four-Jaw Collimator

- ***Doors covered with Li-6 and Cd which absorbs 99% of beam***
- ***Adjustable doors can transmit a beam of any height and width***
- ***Defines beam incident on ion chamber***
- ***Required for neutron polarimetry.***

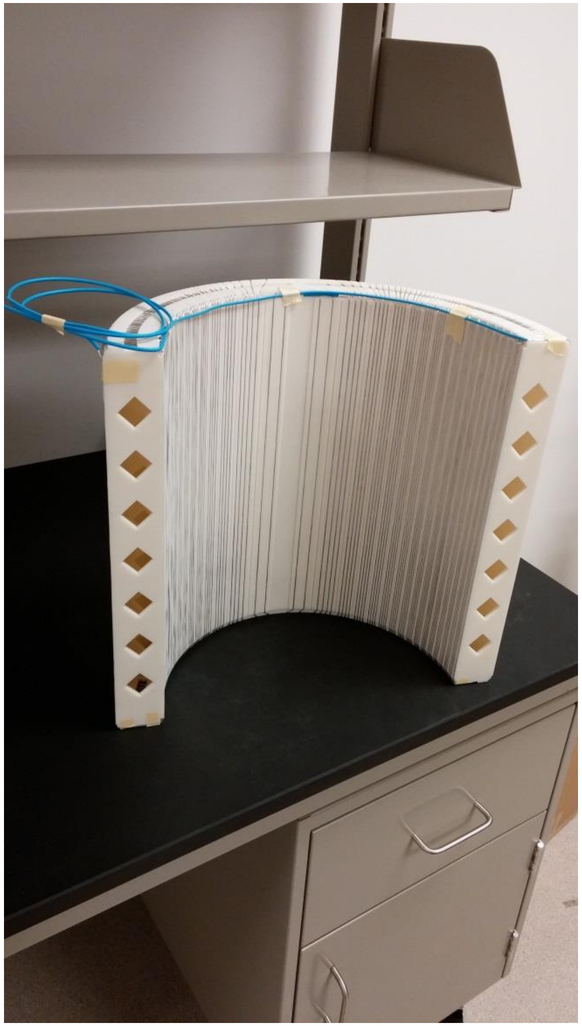


SPIN FLIPPER

Spin Flipper for the n^{3}He Experiment



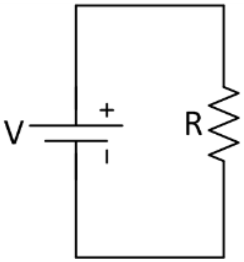
Pictures of Outer Coils



Design of a Cosine Theta Coil

Surface Current Density:

$$\boldsymbol{k}_{in}(\phi) = k_{in} \sin\phi \, \hat{\boldsymbol{z}}$$

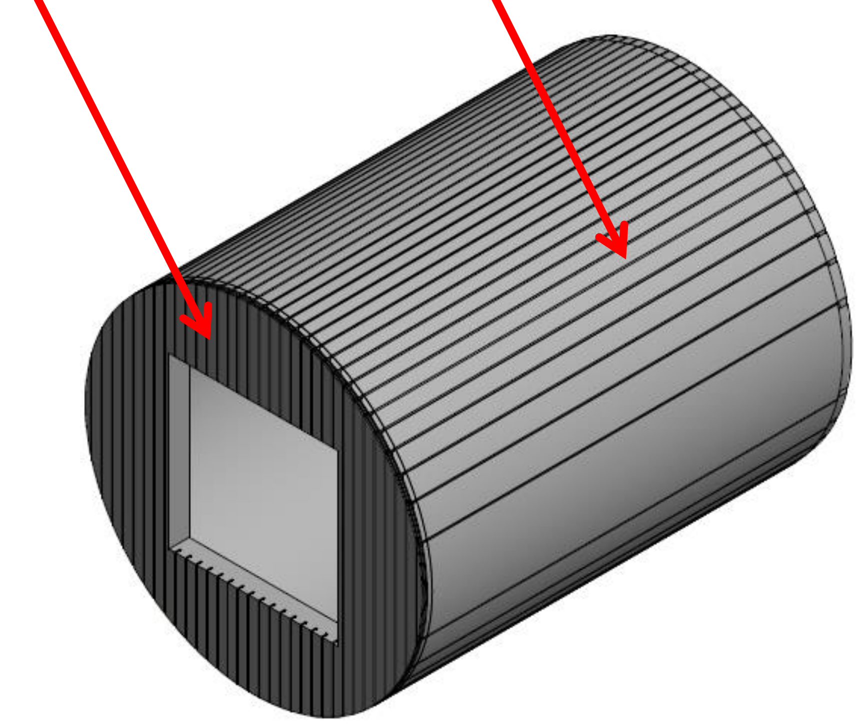


Wires on end caps routed along equi-potentials

Fields determined using theory of a magnetic scalar potential:

$$U_{in} = -\frac{kr}{2}\cos\phi$$

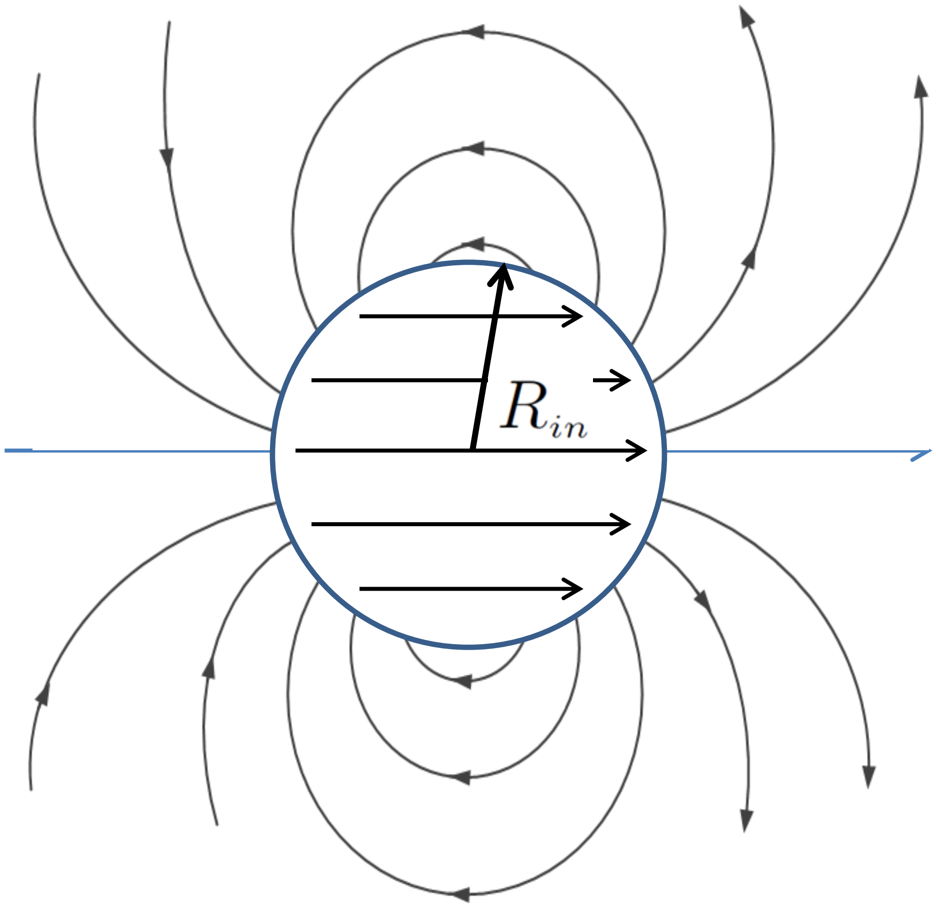
$$U_{out} = \frac{kR_{in}^2}{2r}\cos\phi$$



Fields of a Cosine Theta Coil

$$\boldsymbol{H}_{in} = H_o\hat{\boldsymbol{x}}$$

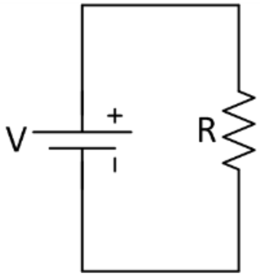
$$\boldsymbol{H}_{out} = \frac{kR_{in}^2}{2r^2}[\cos\phi\,\hat{\boldsymbol{r}} + \sin\phi\,\hat{\boldsymbol{\phi}}]$$



- ***External B-Field is problematic for the n3He experiment***
- ***External B-Field can be the source of false asymmetries***
- ***Need false asymmetries < 10^{-8}***

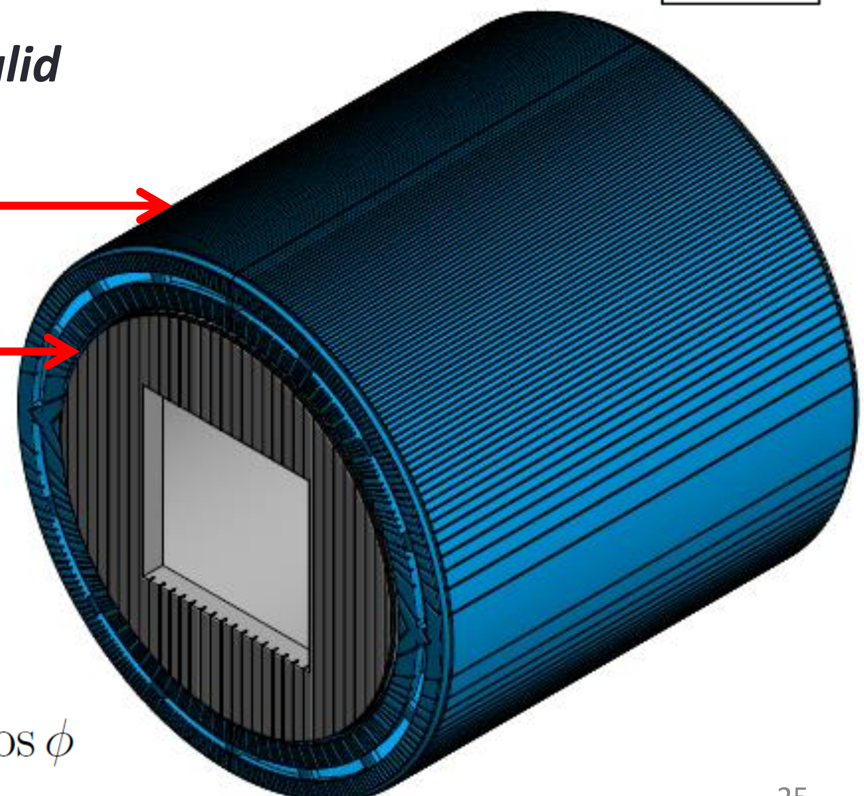
Design of a Double Cosine-Theta Coil

- ***Two Concentric Cosine theta coils***
- ***Currents flow in opposite directions***
- ***Approximation of infinite coil still valid***



$$\boldsymbol{k}_{out}(\phi) = -k' \sin\phi\, \hat{\boldsymbol{z}}$$

$$\boldsymbol{k}_{in}(\phi) = k \sin\phi\, \hat{\boldsymbol{z}}$$



$$U_{in}(r,\phi) = -H_{rf}\, r\cos\phi$$

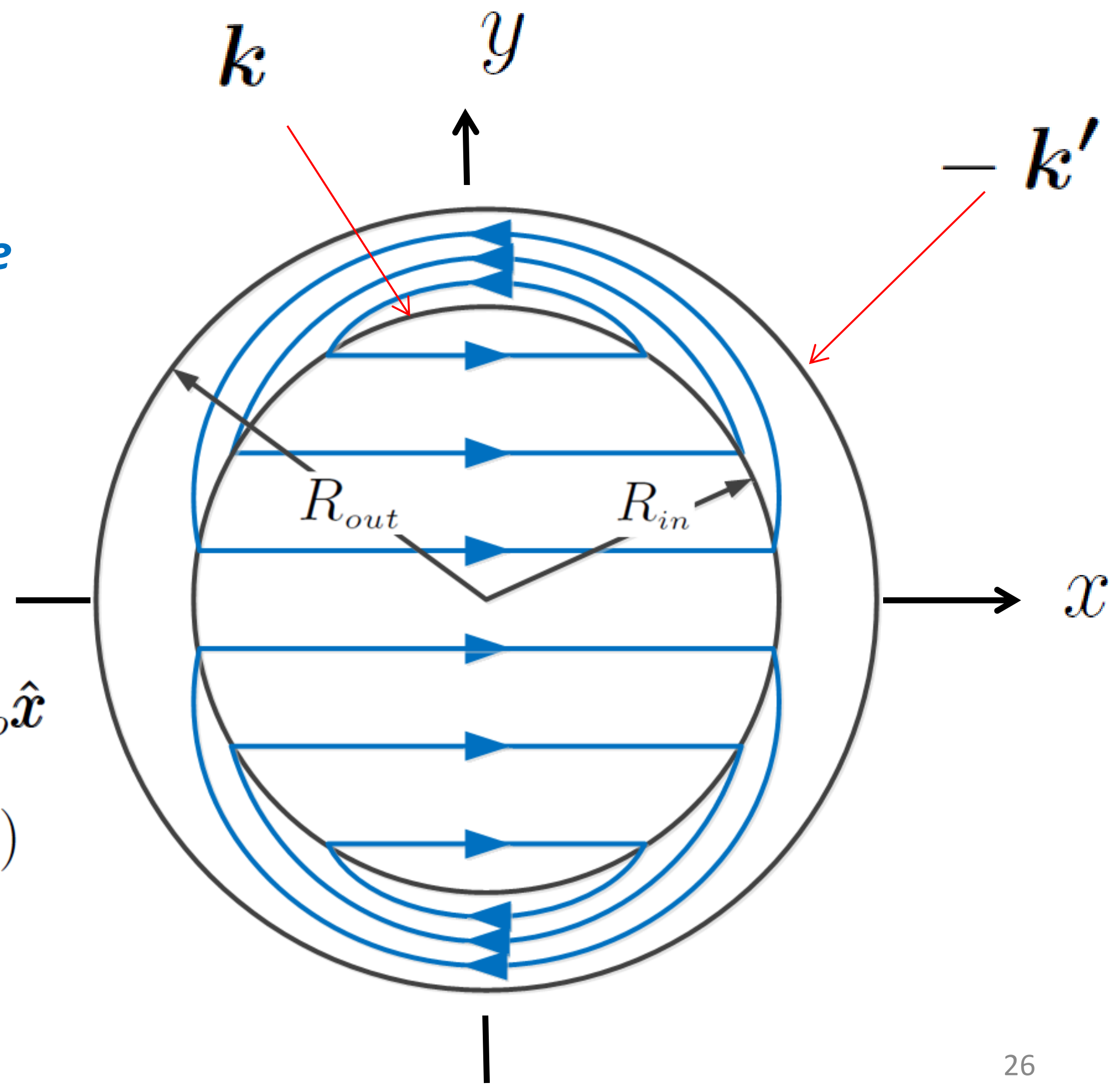
$$U_{out}(r,\phi) = \left[\frac{R_{in}^2}{R_{out}^2 - R_{in}^2}\right] H_{rf} \left[r + \frac{R_{out}^2}{r}\right]\cos\phi$$

Fields of a Double Cosine Theta Coil

--A way to produce a self-contained uniform magnetic field that doesn't interfere with other components in the experiment.

--No False asymmetries from Spin Flipper

$r < R_{in}$	$\boldsymbol{H}_{in} = H_o \hat{\boldsymbol{x}}$
$R_{in} < r < R_{out}$	$\boldsymbol{H}_{mid}(r, \phi)$
$r > R_{out}$	$\boldsymbol{H}_{ext} = 0$



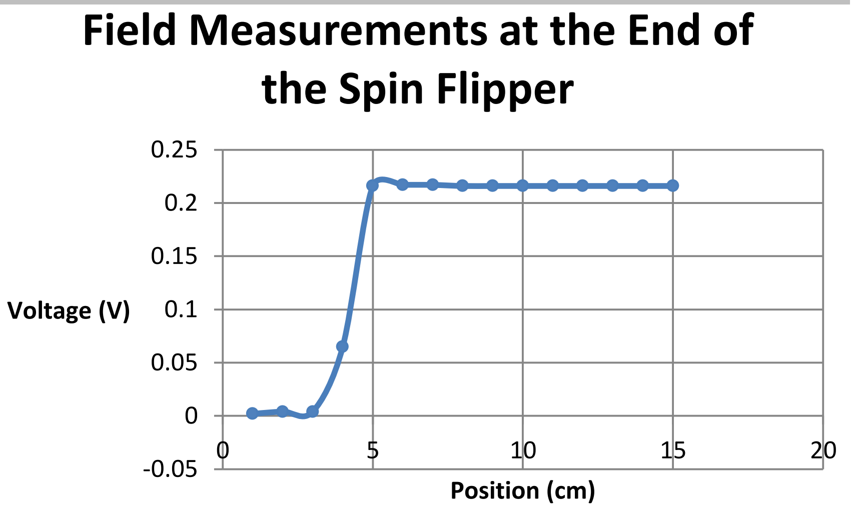
Field Measurements in Spin Flipper

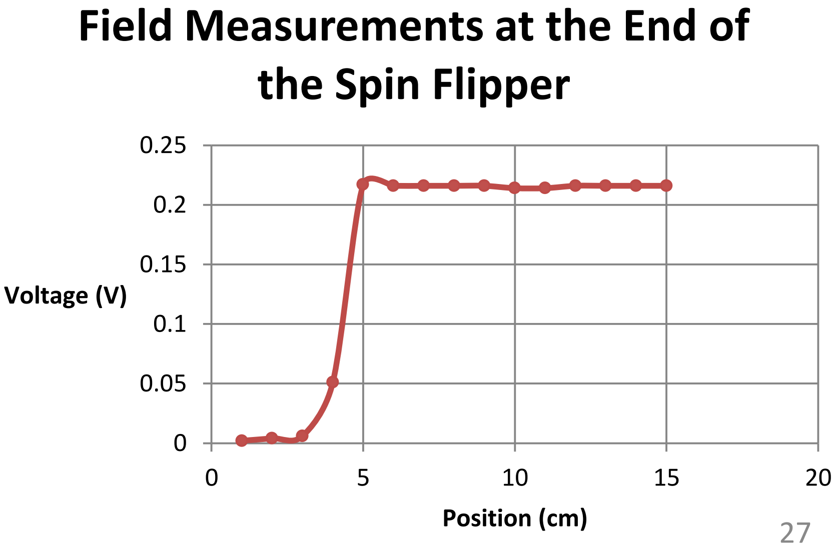
Blue Curve

1	0.002
2	0.004
3	0.004
4	0.065
5	0.216
6	0.217
7	0.217
8	0.216
9	0.216
10	0.216
11	0.216
12	0.216
13	0.216
14	0.216
15	0.216

Red Curve

1	0.002
2	0.004
3	0.006
4	0.051
5	0.217
6	0.216
7	0.216
8	0.216
9	0.216
10	0.214
11	0.214
12	0.216
13	0.216
14	0.216
15	0.216





Inductance of Cosine-Theta Coils

- ***More than one way to calculate inductance of cosine-theta coils:***

  1. ***Magnetic Flux Calculation:*** $$\Phi = LI$$
  2. ***Stored energy in the Magnetic Field:*** $$E = \frac{\mu_o}{2} \int |\boldsymbol{H}|^2 dv$$

- ***Cosine theta coil:*** $$L = \frac{\mu_o \pi z_o}{32} N^2$$

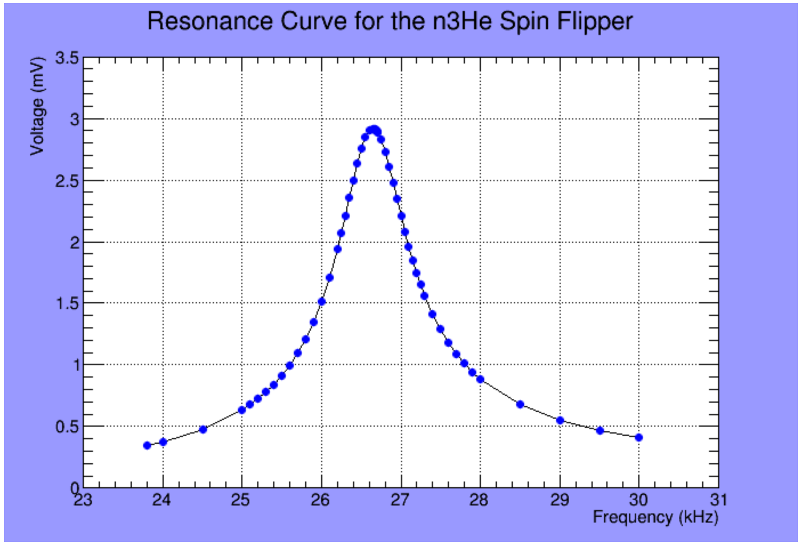
- ***Double cosine theta coil:*** $$L = \frac{\mu_o \pi z_o}{32} N_1 N_2 \left[\frac{R_{out}}{R_{in}} - \frac{R_{in}}{R_{out}} \right]$$

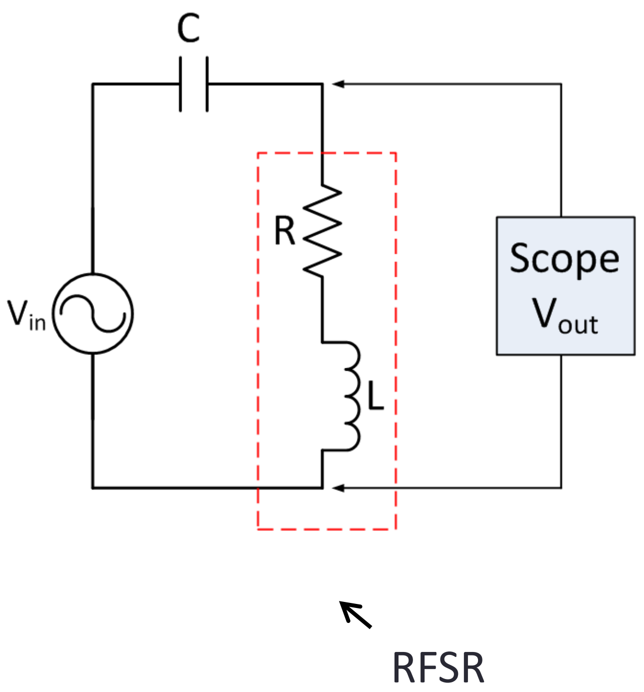
$$L = \frac{5}{16} \mu_o \pi z_o N^2 \longrightarrow L = 2.01\,\mathrm{mH}$$

Double Cosine Theta Coil as RCL Circuit

- *Choose capacitor so that the circuit resonance is at the Larmor Frequency*

f = 26.655 +/- 0.005 kHz





R = 9.11 ohms

L = 2.01 mH

C = 17.70 nF

Measure L using: $C = 1/\omega_L^2 L$

Two-State Spin Magnetic Resonance

- ***Problem solved in the interaction picture***

- ***Two-state problem is exactly solvable (See Sakurai)***

$$i\hbar \begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \begin{bmatrix} E_1 & Ve^{i\omega t} \\ Ve^{i\omega t} & E_2 \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

- ***Solve coupled DE's for probability amplitudes***

- ***Solution requires initial spins all pointing in the same direction***

$$C_1(0) = 1 \qquad C_2(0) = 0$$

$$\boxed{P(t) = \frac{4\omega_F^2}{4\omega_F^2 + (\omega_L - \omega_{rf})^2} \sin^2\left[\sqrt{\omega_F^2 + \frac{(\omega_L - \omega_{rf})^2}{4}}\, t\right]}$$

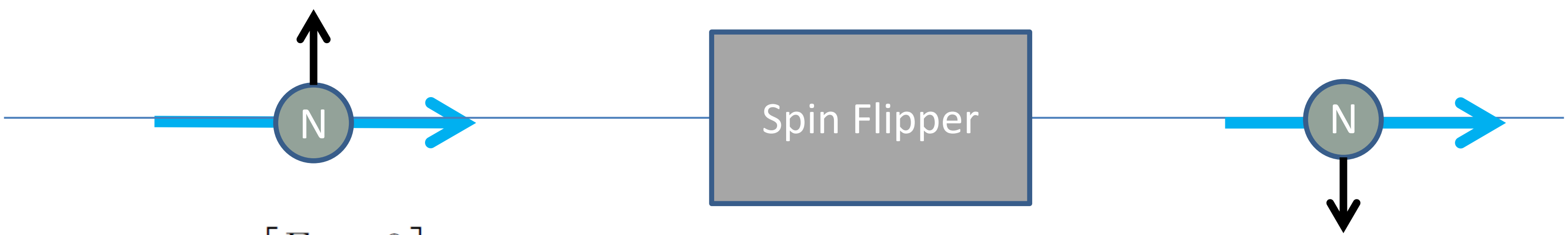
$$P(t) = |C_1(t)|^2 = 1 - |C_2(t)|^2$$

Spin Magnetic Resonance with the Spin Flipper

Drive the Spin Flipper at the Larmor frequency of neutrons in the 10 Gauss B-field

$$\omega_L \equiv \gamma_n B_o$$

$$\boldsymbol{B} = B_o \hat{\boldsymbol{z}} \qquad \mathbf{B} = B_o \hat{\boldsymbol{z}} + B_{rf} \cdot \cos \omega_L t \, \hat{\boldsymbol{x}} \qquad \boldsymbol{B} = B_o \hat{\boldsymbol{z}}$$



$$H = \begin{bmatrix} E & 0 \\ 0 & -E \end{bmatrix} \qquad H \rightarrow \begin{bmatrix} E & V_o e^{i\omega t} \\ V_o e^{-i\omega t} & E \end{bmatrix}$$

$$|\psi\rangle_{up} \qquad |\psi\rangle_{down}$$

- ***Perturbed field inside the spin flipper rotates the state ket***

SMR For a Mixed Ensemble

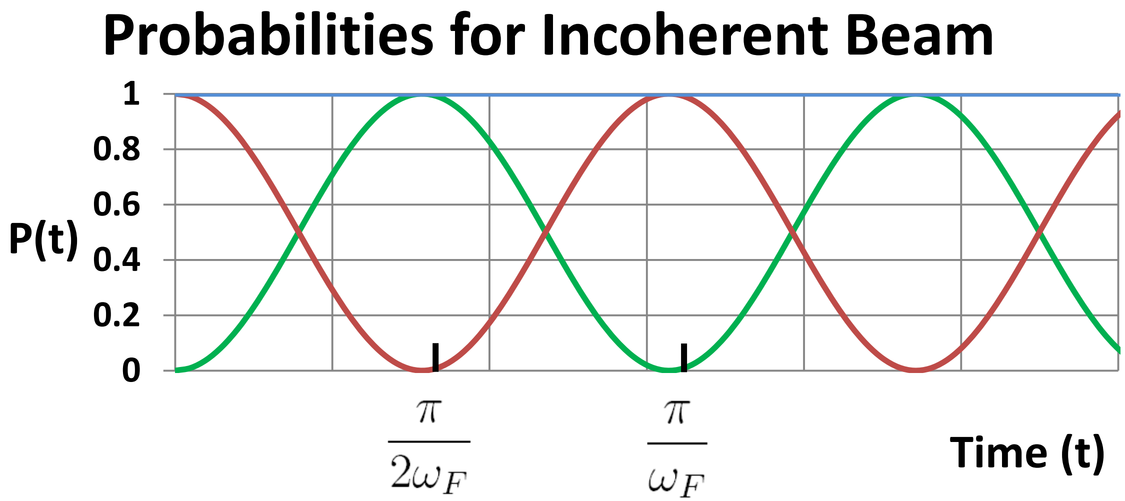
$$\rho = \ .95\ |S_z+\rangle\langle S_z+| + \ .025\ |S_z+\rangle\langle S_z+| + \ .025\ |S_z-\rangle\langle S_z-|$$

- *Solve the SMR problem 3 times –once for each fractional population*

$.95 \ \rightarrow |S_z+\rangle$

$.025 \ \rightarrow |S_z+\rangle$

$.025 \ \rightarrow |S_z-\rangle$



- *Probabilities for transitions of incoherent components oscillate out-of-phase. No contribution to SMR problem*

SMR for a Mixed Ensemble (II)

$$\rho = \textcolor{red}{\mathbf{.95}}\ |S_z+\rangle\langle S_z+| + \textcolor{red}{\mathbf{.025}}\ |S_z+\rangle\langle S_z+| + \textcolor{red}{\mathbf{.025}}\ |S_z-\rangle\langle S_z-|$$

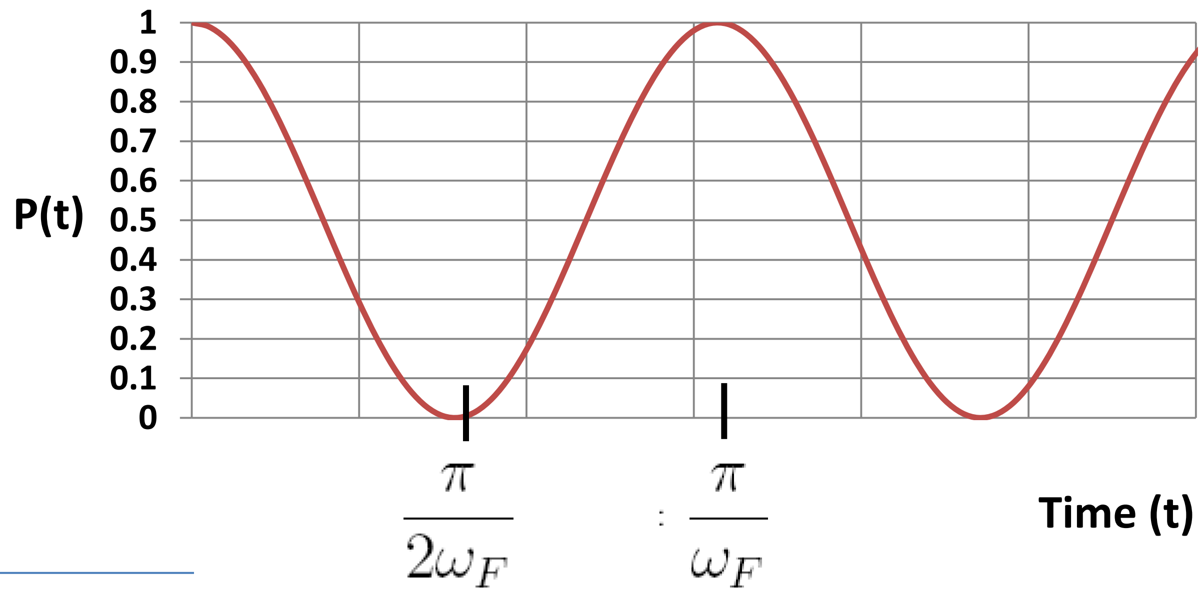
$$\rho = \begin{bmatrix} .975 & 0 \\ 0 & .025 \end{bmatrix}$$

$$\langle S_x \rangle = Tr[\rho S_x] = 0$$

$$\langle S_y \rangle = Tr[\rho S_z] = 0$$

$$\langle S_z \rangle = Tr[\rho S_z] = 0.95\,\frac{\hbar}{2}$$

Probability for Polarized Beam



Flipping Neutrons with a Wavelength Spectrum 2.5 – 6.5 A

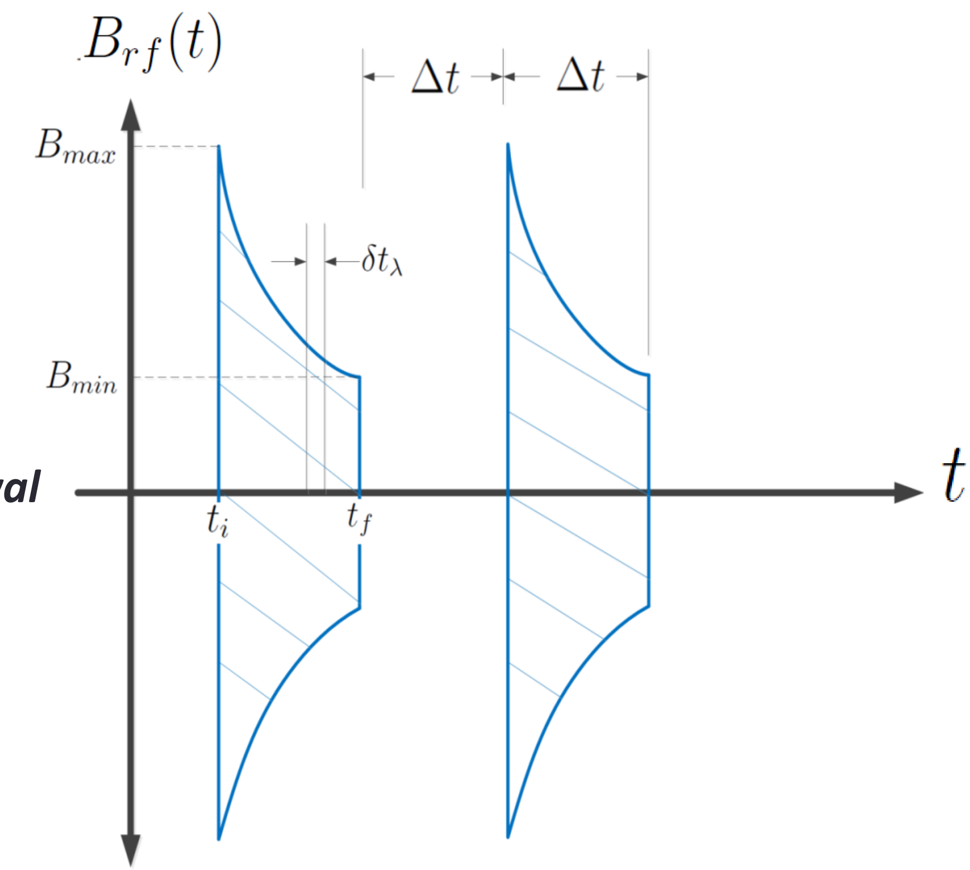
- ***Oscillating RF field only works for a single wavelength***

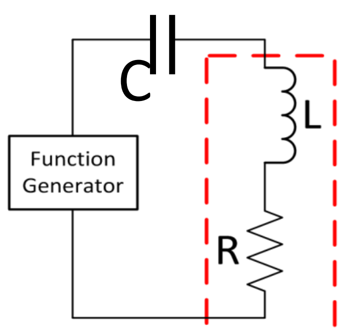
$$\Delta T = t_f - t_i = 16.67\,\mathrm{ms}$$

- ***To flip all neutrons in the beam, add a 1/t time dependence to the RF envelope***

$$B_{rf}(t) = \left[\frac{B_{max}}{1+\alpha t}\right] e^{i\omega_L t}$$

- ***Energy of neutrons in the beam is function of TOF with well defined arrival times***





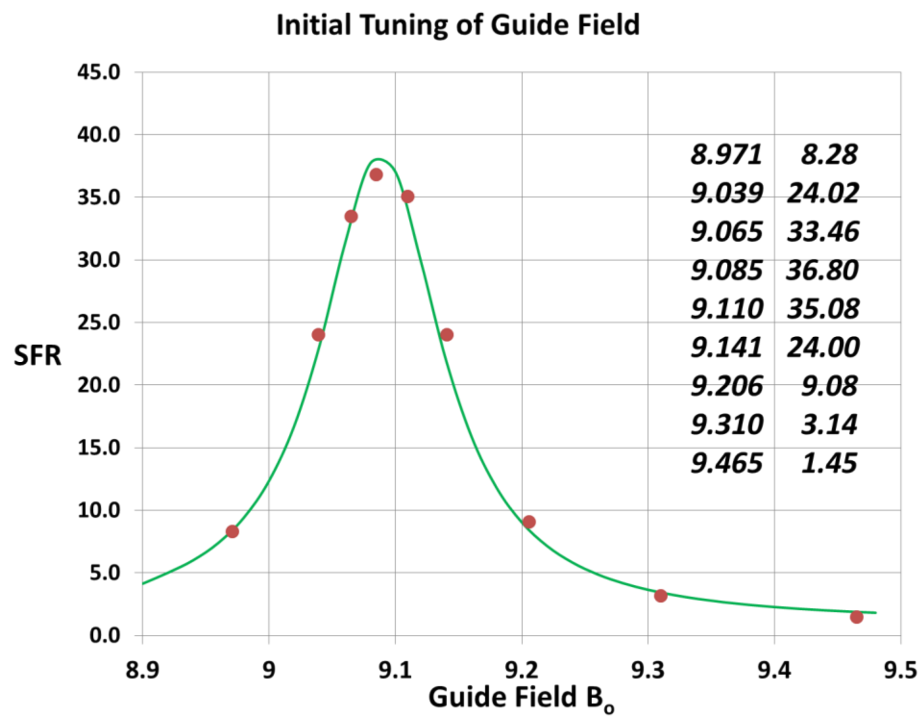
Initial Tuning of the Guide Field

$$P(t) = \frac{4\omega_F^2}{4\omega_F^2 + (\omega_L - \omega_{rf})^2} \sin^2\left[\sqrt{\omega_F^2 + \frac{(\omega_L - \omega_{rf})^2}{4}}\, t\right]$$

$$\mathcal{Q}(\lambda) \longrightarrow \frac{1 + P_n(\lambda)}{1 + \alpha P_n(\lambda)}$$

Spin flip ratio as a function of the guide field:

$$\boxed{\langle \mathcal{Q}(B_o)\rangle = \frac{4\omega_F^2 + (\gamma_n B_o - \omega_{rf})^2}{4 y_n \omega_F^2 + (\gamma_n B_o - \omega_{rf})^2}}$$



$$y_n = \frac{1 - \langle P_n\rangle}{1 + \langle P_n\rangle} \qquad \lambda \sim 4.75\text{Å}$$

Initial Tuning the RF B-Field

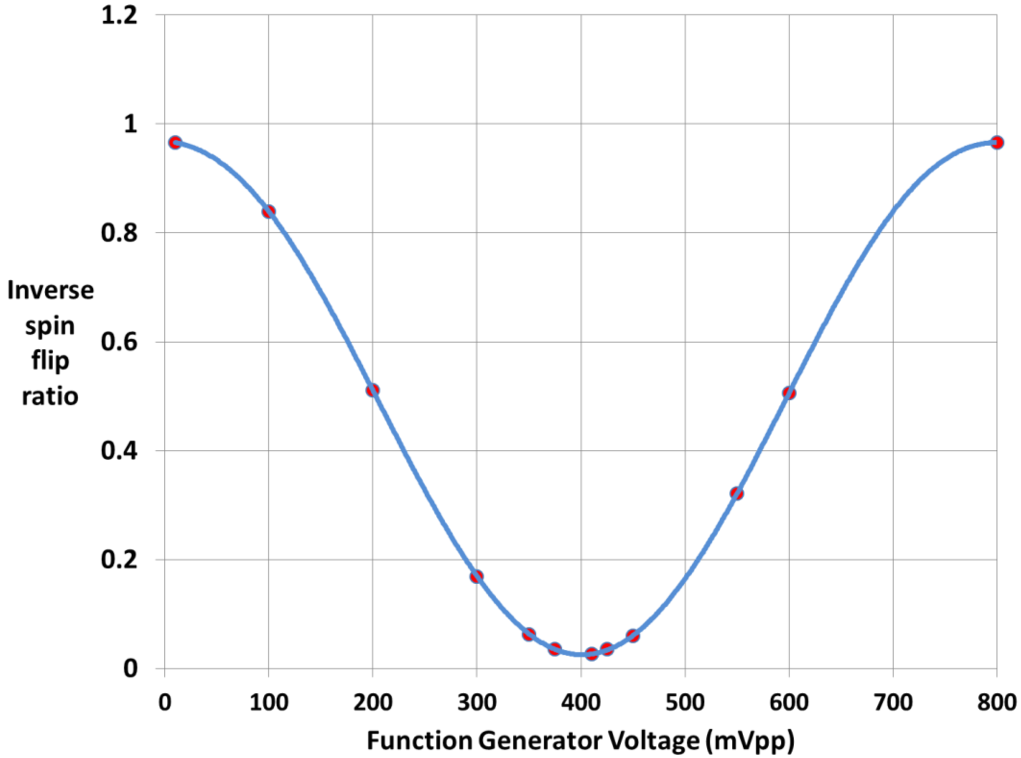
$$P(t) = \frac{4\omega_F^2}{4\omega_F^2 + (\omega_L - \omega_{rf})^2} \sin^2\left[\sqrt{\omega_F^2 + \frac{(\omega_L - \omega_{rf})^2}{4}}\, t\right]$$

$$\mathcal{Q}(\lambda) \longrightarrow \frac{1 + P_n(\lambda)}{1 + \alpha P_n(\lambda)}$$

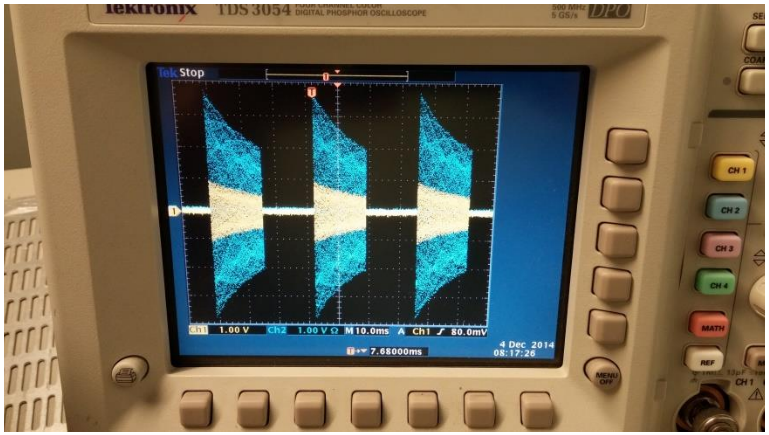
Inverse spin flip ratio versus the spin flipper RF field:

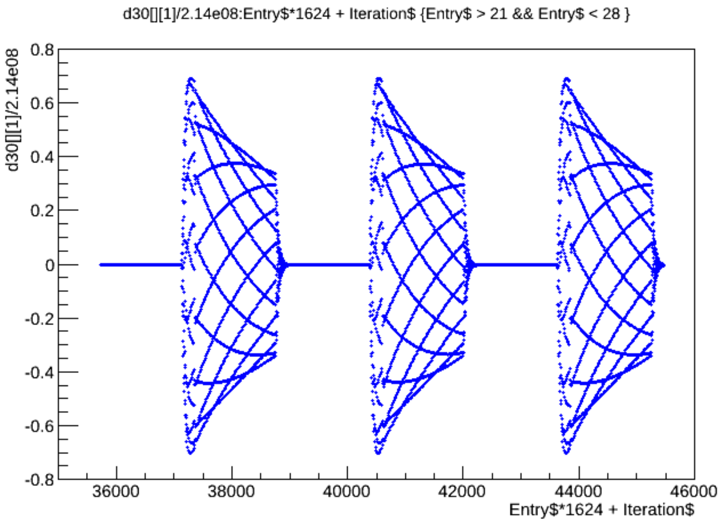
$$\boxed{\langle \mathcal{Q} \rangle^{-1} = \cos^2 \omega_F t + y_n \sin^2 \omega_F t}$$

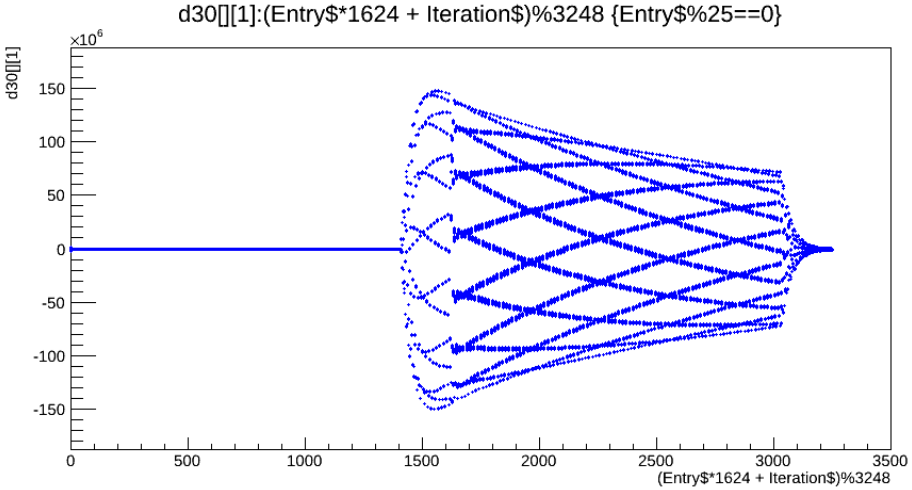
$$\omega_F t = m\pi/2 \quad m = 1, 3, 5 \ldots$$



Spin Flipper Signal Read by DAQ



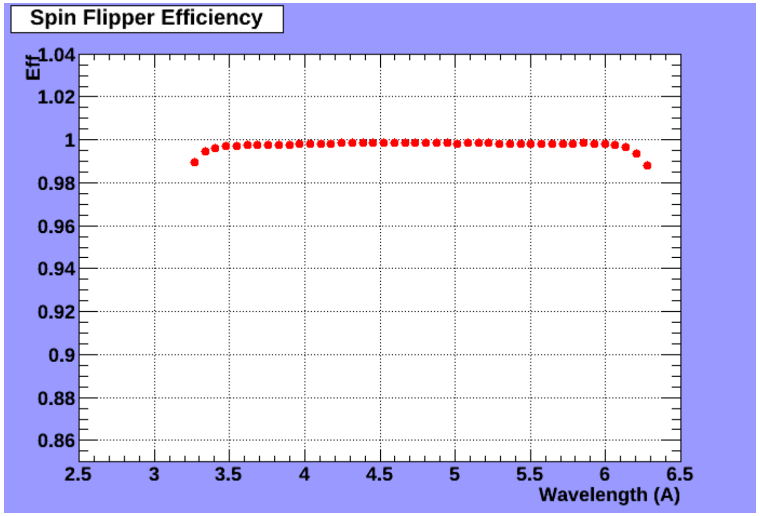


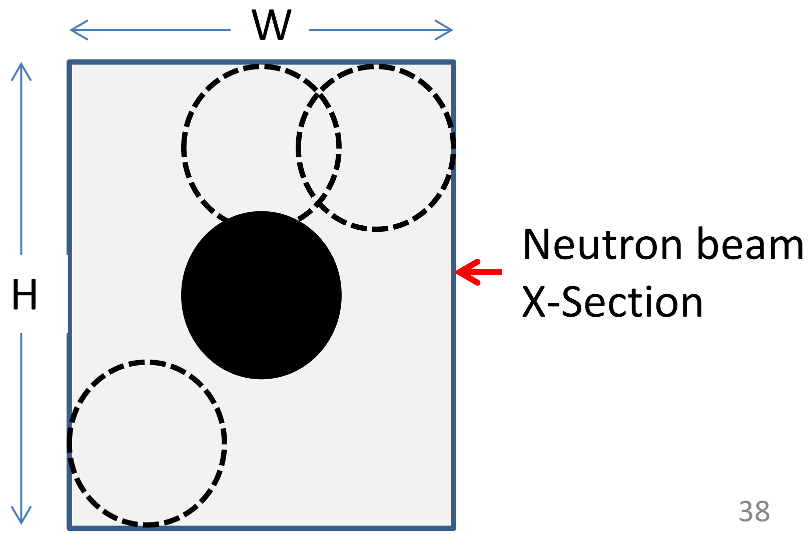


- ***Signal read by the DAQ shows distortion of the signal at the beginning and end of the pulse.***
- ***Vertical line at T=1624 represents DAQ reset to measure next neutron pulse.***

Summary of Spin Flipper Capabilities

- ***A new technology for flipping neutron spins***
- ***Superior operation compared to NPDGamma spin flipper---reaches the limit of efficiency***
- ***Can flip both longitudinal and transverse neutron spin orientations***
- ***Spin flip efficiency is uniform over all parts of the beam***
- ***Should become the industry standard***





Neutron Polarimetry

Neutron Polarimetry

Essential Component of the n^3He experiment.
On a regular basis need to check that:

The neutron beam is what we think it is:

- *Measure beam polarization:* $P_n(\lambda)$

The spin flipper is flipping the way we think it should.

- *Measure spin flipper Efficiency:* $\epsilon_{sf}(\lambda)$

- *Keep guide field tuned to optimize spin flipper operation*

Polarimetry Results

$$A_p = \frac{A_{obs}}{\langle P_n \rangle \cdot \langle \epsilon_{sf} \rangle}$$

$$\boxed{\begin{aligned} \langle P_n \rangle &= 0.936 \pm 0.0018 \\ \langle \epsilon_{sf} \rangle &= 0.9979 \pm .00091 \end{aligned}}$$

- ***Determine*** $P_n(\lambda)$ ***and*** $\epsilon_{sf}(\lambda)$ ***during neutron polarimetry in the range 3.5 – 6.0 Å***
- ***Average over neutron wavelengths and then average again over each polarimetry***

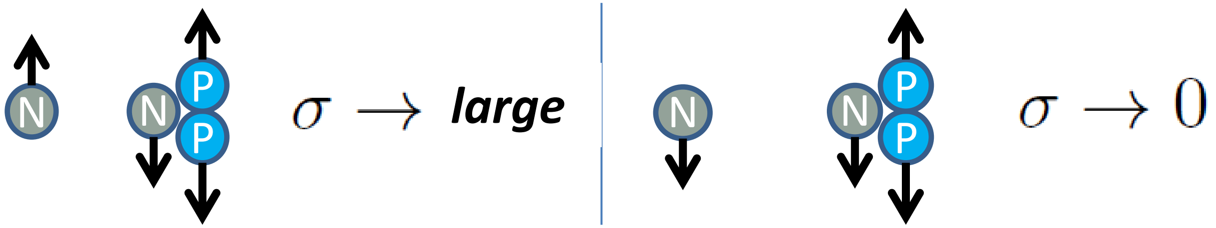
Polarimetry Dates:

----

1. 01/28/2015
2. 02/17/2015
3. 02/25/2015
4. 03/25/2015
5. 04/21/2015
6. 05/20/2015
7. 06/23/2015
8. 08/17/2015
9. 08/24/2015
10. 09/23/2015
11. 10/22/2015
12. 11/30/2015

Properties of a ^{3}He Cell

- ***Filled with*** ^{3}***He, Rb, K, N_2***



$$n = 3.396 \times 10^{25} \text{ atoms}/m^3$$
$$\sigma_o = 5316 \, \text{bn}$$
$$\lambda_o = 1.798 \, A$$
$$\ell = 10.3 \text{ cm}$$

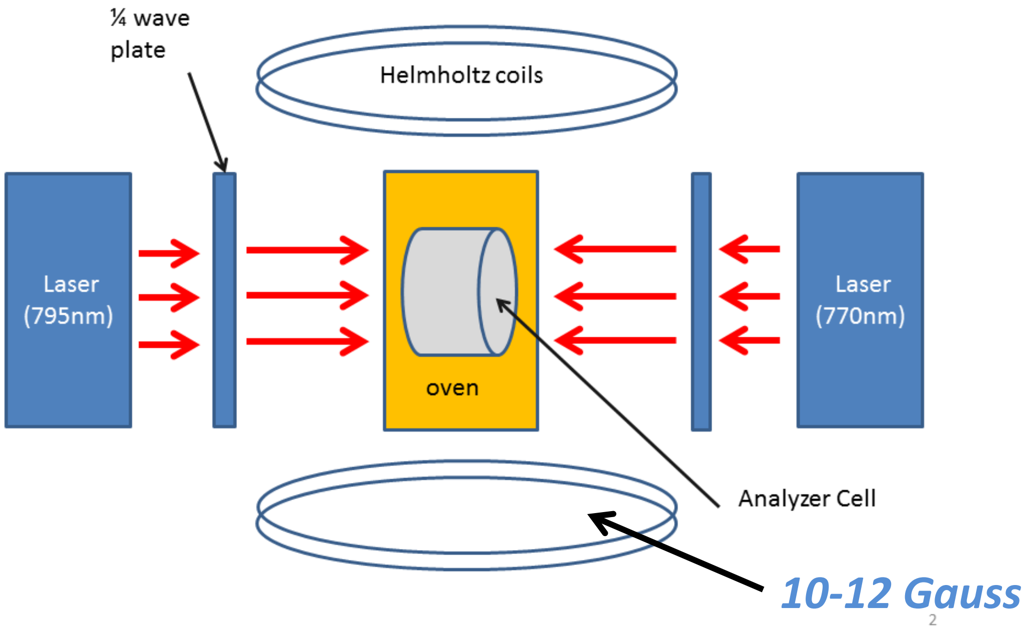
$$\chi \equiv \frac{n\sigma_o\ell}{\lambda_o} = 1.004$$

Cell Thickness

$$T(\lambda) = T_o(\lambda) \exp\left[-\frac{n\sigma_o\lambda\ell}{\lambda_o}\right] \quad \rightarrow T_o(\lambda)\, e^{-\chi\lambda}$$

Hedy Lamarr

Spin Exchange Optical Pumping



- *^{3}He placed in external B-field and heated to 195 C to vaporize Rb and K Metals*

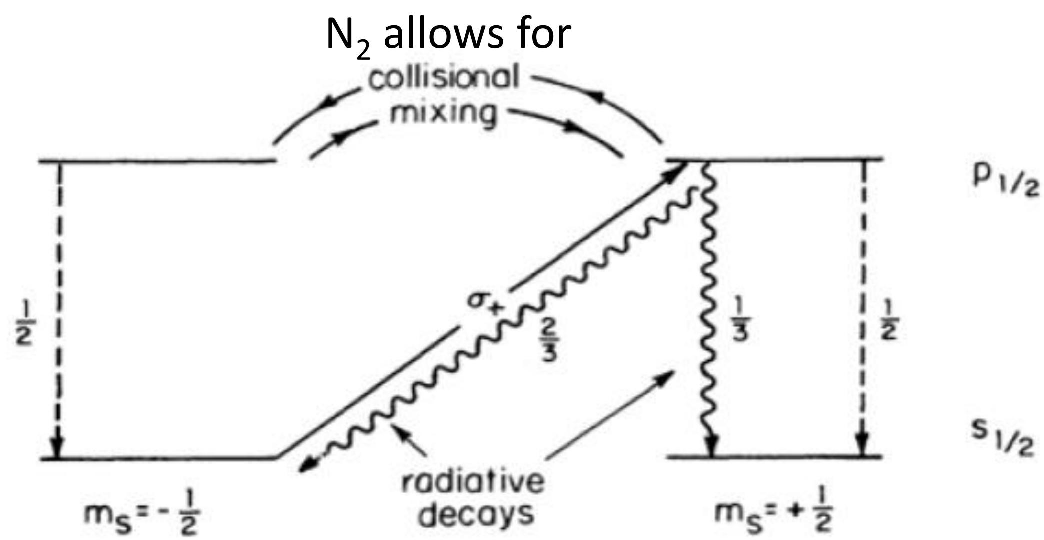
- *Focus infra-red Laser light on cell to polarizes Rb and K vapors*

- *Polarized Rb and K transfer polarization to ^{3}He nuclei thru hyperfine interaction*

- *Cell polarizes overnight*

Optical Pumping of Alkali Metals

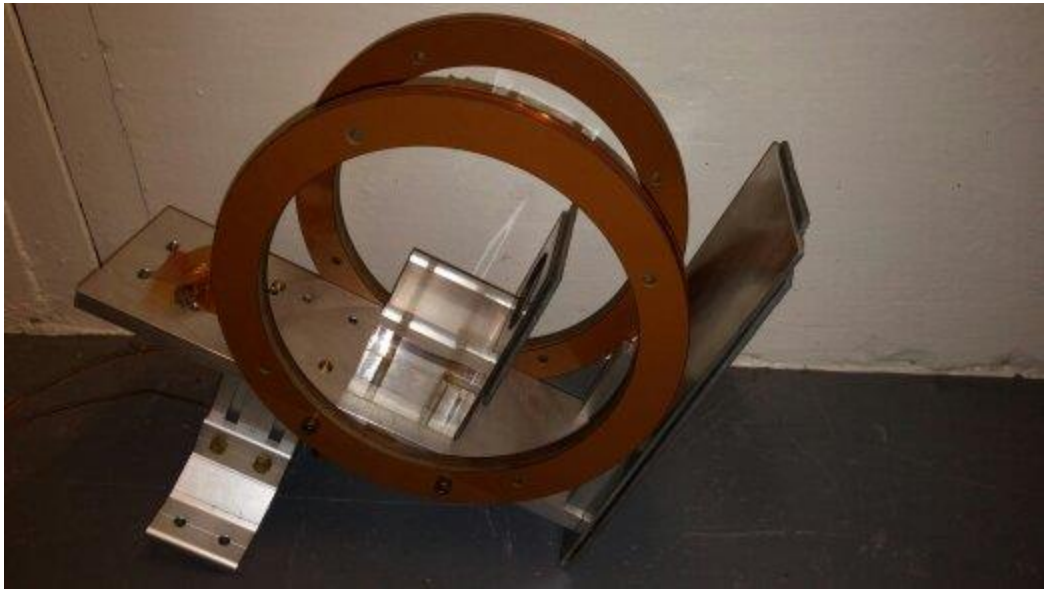
- ***Incident circularly polarized light can only be absorbed by the Ms = -1/2 sub-state***
- ***$P_{1/2}$ state will decay to either $S_{1/2}$ sub-state***
- ***Partial pressure of N_2 induces collisional mixing/increases cell polarization***



$$P_A = \rho_A(+\tfrac{1}{2}) - \rho_A(-\tfrac{1}{2})$$

$$P(t) = \frac{\gamma_{SE} P_A}{\Gamma + \gamma_{SE}} \left[1 - e^{-(\Gamma + \gamma_{SE})t}\right]$$

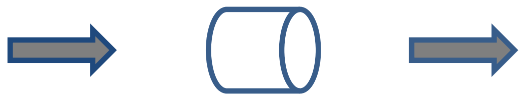
Polarimetry Apparatus/Setup



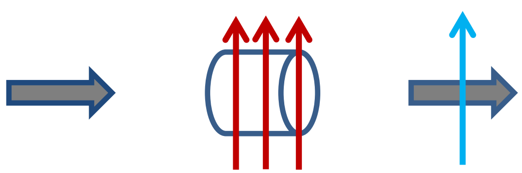
- ***Polarimetry apparatus centered near beam centroid***
- ***Polarized cell sits in acrylic V-Block assembly***

Transmission thru the Analyzer Cell

$T_o(\lambda)$

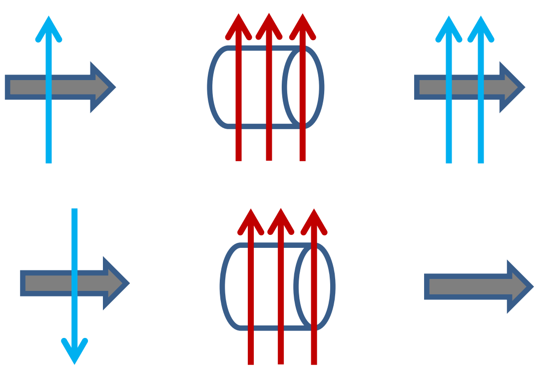


$$T(\lambda) = T_o(\lambda)e^{-\chi\lambda}$$



$$T(\lambda) = T_o(\lambda) \cdot e^{-\chi\lambda} \cosh(\chi\lambda P)$$

Analyzing Power of the ^{3}He cell: $P_n(\lambda) = \tanh(\chi\lambda P)$



Large transmission for parallel spins

$$T(\lambda) = T_o(\lambda) \cdot e^{-\chi\lambda} \left[\cosh(\chi\lambda P) + P_n \sinh(\chi\lambda P)\right]$$

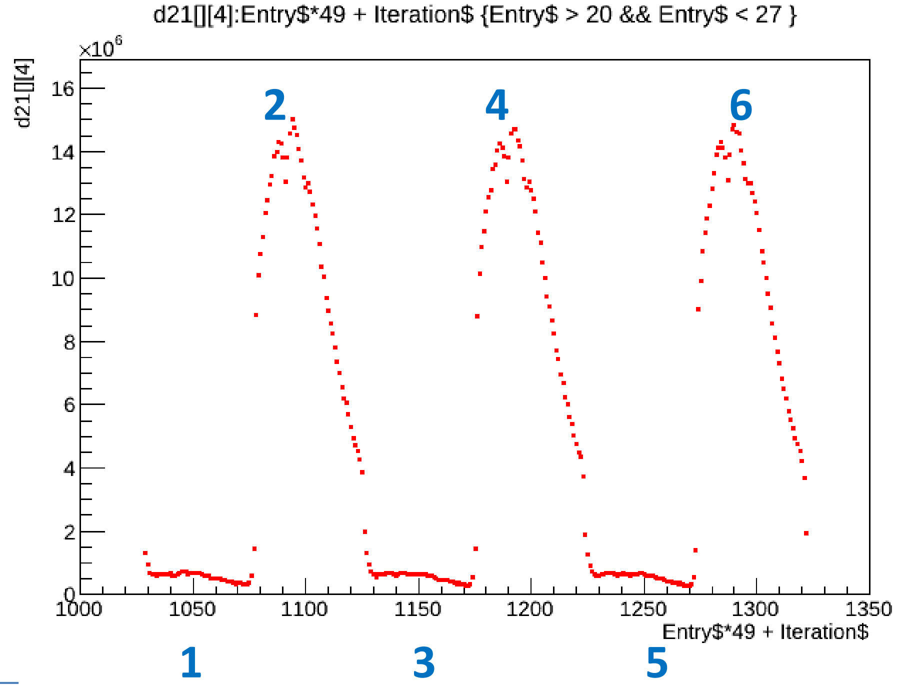
Large absorption for anti-parallel spins

Calculating Neutron Beam Polarization

Polarization determined by measuring beam transmission thru:

- ***Polarized Cell***
- ***Un-polarized Cell***

$$R_1 \equiv \frac{T_{on}}{T_{unp}} \qquad R_2 \equiv \frac{T_{off}}{T_{unp}}$$



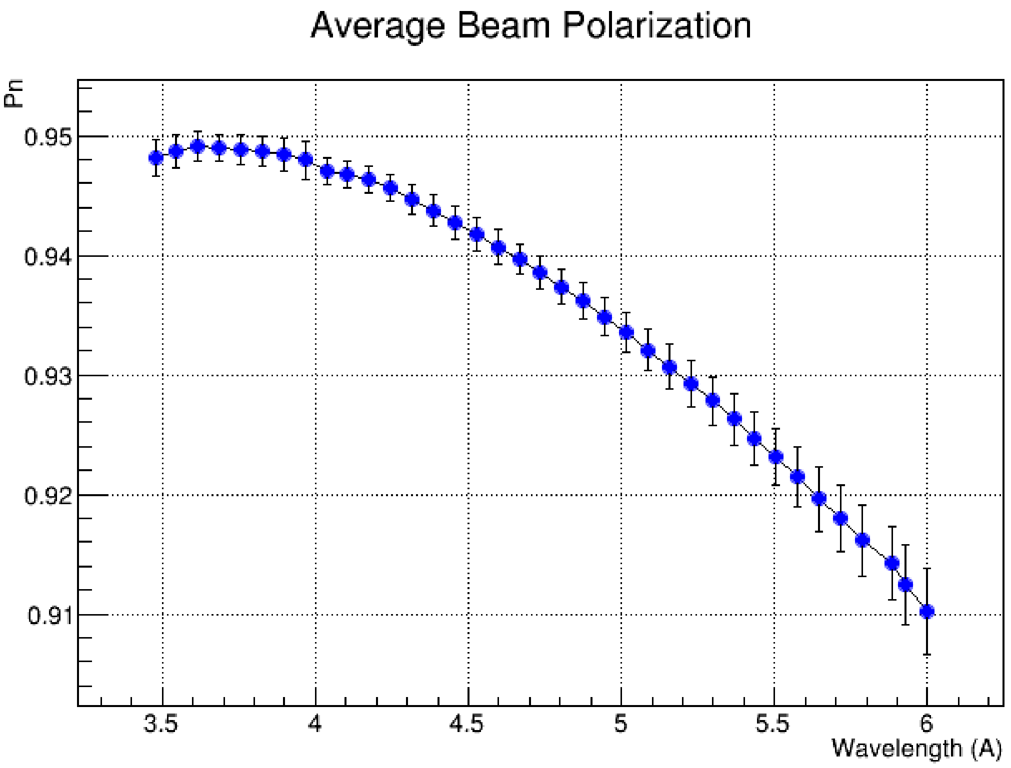
$$P_n(\lambda) = \frac{R_1 - R_2}{\sqrt{[R_2 - (1-2\epsilon_{sf})R_1]^2 - 4\epsilon_{sf}^2}} \rightarrow \boxed{P_n(\lambda) = \frac{R_1 - R_2}{\sqrt{[R_2 + R_1]^2 - 4}}}$$

Neutron Beam Polarization

- ***Polarization determined from 8 independent measurements Feb – Nov 2015***
- ***Beam polarization measured at beam center:***

$$\langle P_n \rangle = 0.936 \pm 0.0018$$

- ***Slope of Beam polarization curve due to supermirror polarizer***

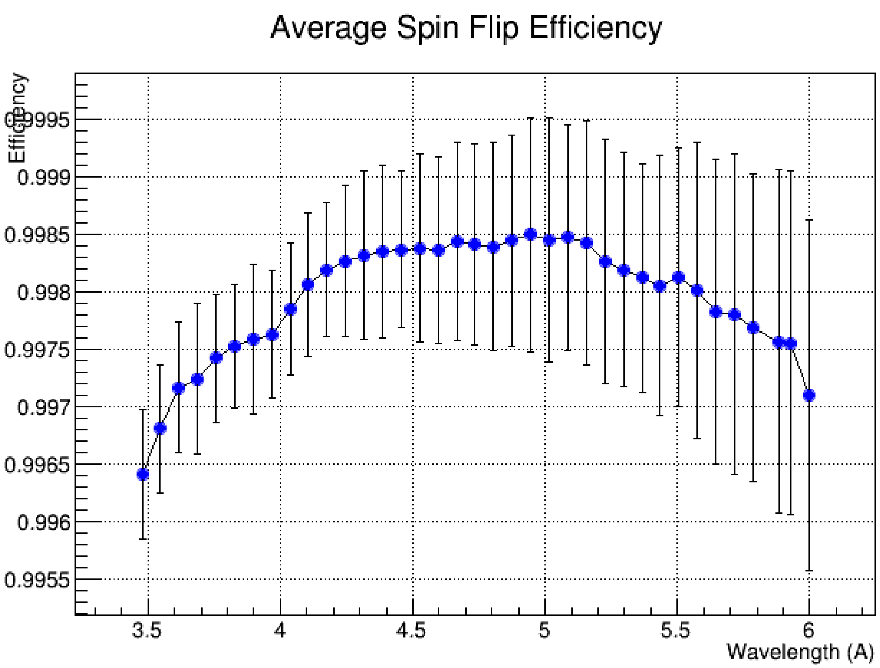


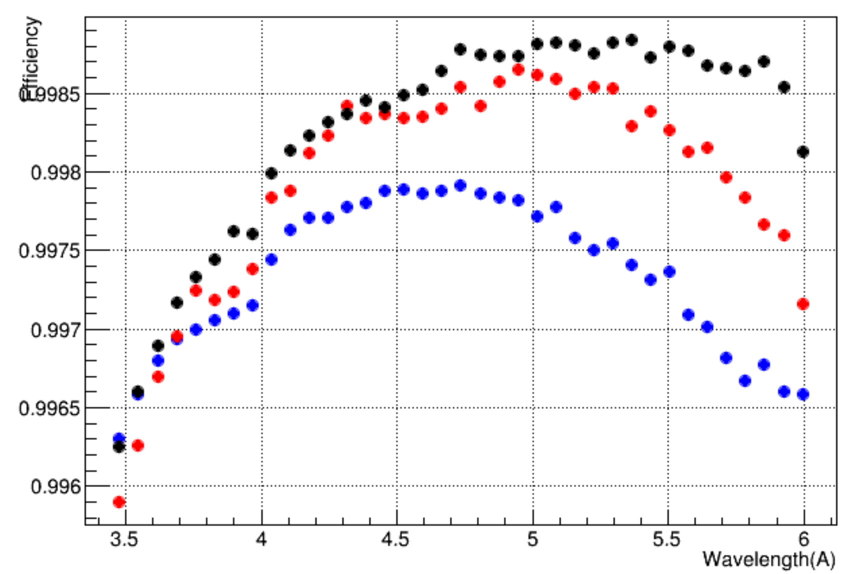
Calculation of Spin Flipper Efficiency

$$\epsilon_{sf} = \frac{1}{2}\left[1 - \frac{P_{on}}{P_{off}}\right]$$

$$\langle \epsilon_{sf} \rangle = 0.9979 \pm .00091$$

- ***Calculate spin flipper efficiency by reversing polarization of the He-3 cell (AFP Flip)***
- ***Off-axis measurements of spin flipper efficiency show no significant changes***





Contributions to the n^3He Experiment

- ***Completed all aspects of the design, construction, testing, and integration of the Spin Flipper***
- ***Completed design and construction of the 4-Jaw collimator***
- ***Completed design of the V-Block/Sled alignment hardware***
- ***Completed design of two XY scanner mounts for location of Beam Centroid***
- ***Major contributor to the design/construction/implementation of the alignment laser beam***
- ***Major role in completing alignment of the guide field in the cave***
- ***Presentations on the spin flipper for APS and ACNS***
- ***Performed all polarimetry measurements for the n^3He experiment***

n^{3}He Collaboration

n^{3}He Collaboration

Pil-Neo Seo[1], Michele Viviani[2], Seppo Pentillä[3], David Bowman[3], Vince Cianciolo[3], Jack Thomison[3], Chris Crawford[4], Latiful Kabir[4], Aaron Sprow[4], Ivan Novikov[5], Michael Gericke[6], Mark McCrea[6], Carlos Olguin[6], Libertad Baron[7], John Calarco[8], Vladimir Gudkov[9], Young-Ho Song[9], Nadia Fomin[10], Geoff Greene[10], Christopher Hayes[10], Irakli Garishvili[10], Eric Plemons[10], Josh Hamblen[11], S. Baessler[12]

[1]Duke University, Triangle Universities Nuclear Laboratory
[2]Istituto Nazionale di Fisica Nucleare, Sezione di Pisa
[3]Oak Ridge National Laboratory
[4]University of Kentucky
[5]Western Kentucky University
[6]University of Manitoba
[7]Universidad Nacional Autónoma de México
[8]University of New Hampshire
[9]University of South Carolina
[10]Univeristy of Tennessee
[11]Univeristy of Tennessee at Chattanooga
[12]University of Virginia