Magnet Fit Procedure

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9:37 PM

* we are fitting field measurements from different orientations of the probe to the following model:

$\vec{m}^{\alpha\beta} = R S^{\alpha} \vec{b}^{\beta} + \vec{m}_0$ $\quad \vec{m}^{\alpha\beta}$ = meas, S^{α} = sym, R = resp, $\vec{b}^{\beta}$ = field

$m_i^{\alpha\beta} = R_{ij} S_{jk}^{\alpha} b_k^{\beta} + m_{0i}$ $\quad$ i, j, k are components

let $\beta = 1$: only fit to one magnetic field value.

We want to fit for R, $\vec{b}$, and $\vec{m}_0$

the independent variabe is S^{α}, describing the symmetry orientation of the probe, and $\vec{m}^{\alpha}$ is the dependent variable (data).

In matrix form, ($\otimes$ is the Kronecker product)

$$
{}_{16}\begin{pmatrix} m_1^1 & m_2^1 & m_3^1 \\ m_1^2 & m_2^2 & m_3^2 \\ \vdots & & \\ m_{16}^1 & m_{16}^2 & m_{16}^3 \end{pmatrix}_3 = {}_{16}\begin{pmatrix} S_{11}^1 & S_{12}^1 & S_{13}^1 & S_{21}^1 & S_{22}^1 & S_{23}^1 & S_{31}^1 & S_{32}^1 & S_{33}^1 & 1 \\ S_{11}^1 & S_{12} & S_{13} & S_{21} & S_{22} & S_{23} & S_{31} & S_{32} & S_{33} & 1 \\ \vdots & & \vdots & & \vdots & & & \vdots & & \\ S_{11}^{16} & S_{12}^{16} & S_{13}^{16} & S_{21}^{16} & S_{22}^{16} & S_{23}^{16} & S_{31}^{16} & S_{32}^{16} & S_{33}^{16} & 1 \end{pmatrix} {}_{10}\left(\begin{array}{c} \begin{pmatrix} R_{11} & R_{21} & R_{31} \\ R_{12} & R_{22} & R_{32} \\ R_{13} & R_{23} & R_{33} \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \\ \hline m_0^1 \quad m_0^2 \quad m_0^3 \end{array} \right)_3
$$

or $M = (S|1)\begin{pmatrix} R^T \otimes b \\ \hline m \end{pmatrix}$.

First let ${}_{16}(S|1)_{10} = {}_{16}U_{16}\,\Sigma_{10}\,V^T_{10}$ (singular value decomposition)

Then ${}_{10}\begin{pmatrix} R^T \otimes b \\ \hline m \end{pmatrix}_3 = {}_{10}V_{10}\,\Sigma^{-1}_{16}\,U^T_{16}\,M_3$ $\qquad U^TU = 1 \quad V^TV = 1$

where $\Sigma = \begin{vmatrix} \sigma_1 & & \\ & \sigma_2 & 0 \end{vmatrix}$, $\Sigma^{-1} = \begin{vmatrix} \sigma_1^{-1} & & 0 \cdots 0 \\ & \sigma_2^{-1} & 0 \cdots 0 \end{vmatrix}$

where $\Sigma = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & 0 \\ 0 & & \ddots & \\ & & & \sigma_{10} \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$, $\Sigma^{-1} = \begin{pmatrix} \sigma_1^{-1} & & & & 0 & \cdots & 0 \\ & \sigma_2^{-1} & & & 0 & \cdots & 0 \\ & & \ddots & & \vdots & & \vdots \\ & & & \sigma_{10}^{-1} & 0 & \cdots & 0 \end{pmatrix}$

Pick off the value of $\vec{m}$, and you are left with the matrix

$$ {}_9T_3 = {}_3R_3 \otimes {}_3b_1 = \begin{pmatrix} R_{11} & R_{21} & R_{31} \\ R_{12} & R_{22} & R_{32} \\ R_{13} & R_{23} & R_{33} \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} R_{11}b_1 & R_{21}b_1 & R_{31}b_1 \\ R_{11}b_2 & R_{21}b_2 & R_{31}b_2 \\ R_{11}b_3 & R_{21}b_3 & R_{31}b_3 \\ R_{12}b_1 & R_{22}b_1 & R_{32}b_1 \\ R_{12}b_2 & R_{22}b_2 & R_{32}b_2 \\ R_{12}b_3 & R_{22}b_3 & R_{32}b_3 \\ R_{13}b_1 & R_{23}b_1 & R_{33}b_1 \\ R_{13}b_2 & R_{23}b_2 & R_{33}b_2 \\ R_{13}b_3 & R_{23}b_3 & R_{33}b_3 \end{pmatrix} $$

Rearrange this matrix into $\mathcal{T} = \mathcal{R}\, b$

$$ \begin{pmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \\ T_{41} & T_{51} & T_{61} \\ T_{42} & T_{52} & T_{62} \\ T_{43} & T_{53} & T_{63} \\ T_{71} & T_{81} & T_{91} \\ T_{72} & T_{82} & T_{92} \\ T_{73} & T_{83} & T_{93} \end{pmatrix} = \begin{pmatrix} R_{11} \\ R_{21} \\ R_{31} \\ R_{12} \\ R_{22} \\ R_{32} \\ R_{13} \\ R_{23} \\ R_{33} \end{pmatrix} (b_1 \; b_2 \; b_3) $$

or $\mathcal{T} = \mathcal{R}\, b$ and again

let $\mathcal{T} = U' \Delta V'^T$ (SVD)

Then $\hat{\mathcal{R}}$ is the column of U' corresponding to the largest eigenvalue in Δ and $\hat{b}$ is the corresponding column of V'.

Both $\hat{\mathcal{R}}$ and $\hat{b}$ are unit vectors still.

Normalize $b = B\hat{b}$ to the know magnitude of the field B,

then $\mathcal{R} = \hat{\mathcal{R}} \cdot \lambda_1 / B$ so that $\tau = \mathcal{R} \lambda_1 b$.

Use $\mathcal{R}$ to construct the matrix

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \quad \text{so that} \quad \vec{m} = R\vec{b} + \vec{m}_0.$$

Given a measurement $\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$, $\vec{b} = R^{-1}(\vec{m} - \vec{m}_0)$.

gives the actual value of the magnetic field along the axes of symmetry.

You must rotate by 45° to get the field in the lab co-ordinate system.