Calculating BL-13 relative neutron spectrum from M1

CHRISTOPHER COPPOLA

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1 Generating the neutron spectrum

In order to generate an accurate simulation of $n+^3$He, we need a way to reproduce the energy spectrum of the neutrons entering the detector. The beam monitor M1 samples the neutron intensity at 100kHz, which we can use to determine this. Since the neutrons come from a pulsed source, the time-of-flight is highly correlated with the energy of the neutron (see Appendix A). If we can make a precise calcluation of the time of arrival of each neutron bin, we can determine their relative intensity, and produce a normalized spectrum.

We will need to make some assumptions in order to produce this calculation. First, we will assume that the width of the neutron pulse at the moderator face is much smaller than the time of flight to our detector. Second, we will ignore path lengthening due to irregular trajectories. In this model, the velocity of a neutron arriving at time t is

$$v(t) = \frac{\Delta x}{\Delta t} \tag{1}$$

So, in order to calculate the velocity, we need the path length as well as the exact time-of-flight. Let's first examine the path length.

2 Path length

We could try to reconstruct the path length from the SNS design documents (Appendix B), but this would assume perfect construction and neglect any physics which would deviate from geometric particle propagation. Fortunately, we are given some precise timing information in the shape of the monitor signal: there are two prominent peaks due to the scattering of neutrons in Aluminum:

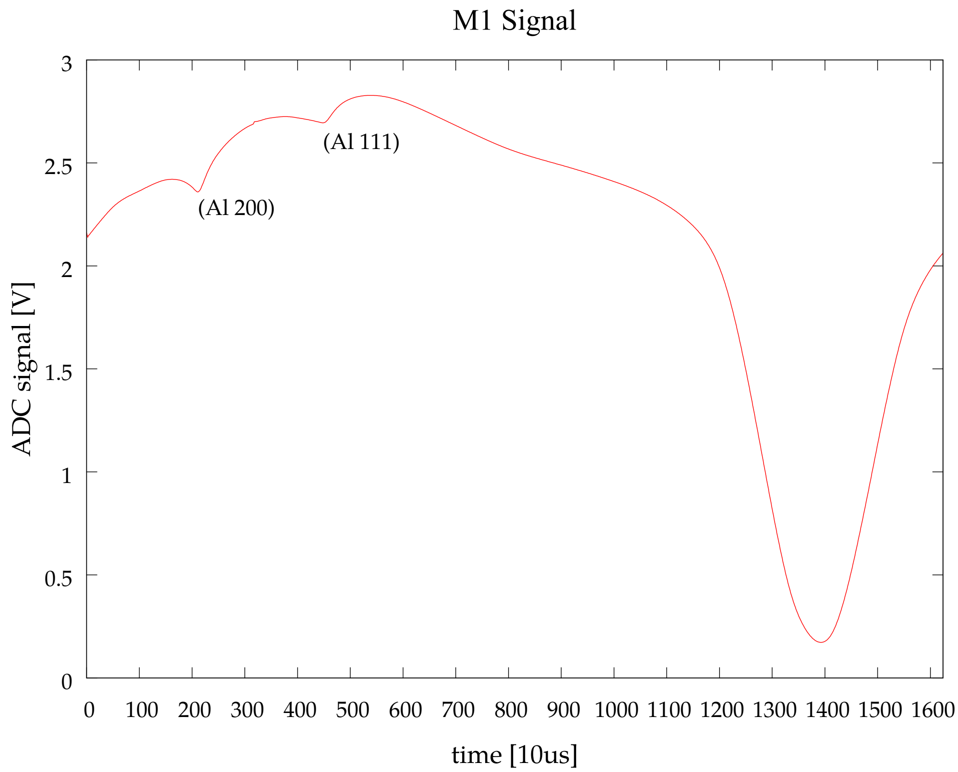


Figure 1: Shape of the neutron pulse in monitor 1.

We know, approximately, the range of energies selected by the choppers. So we can guess which plane spacings in Al that these two peaks correspond to:

$$\frac{\lambda_1}{2} = \frac{a}{\sqrt{\vec{h_1}^2 + \vec{k_1}^2 + \vec{l_1}^2}} = \frac{4.0495 \cdot 10^{-9} m}{\sqrt{1+1+1}}; \qquad \lambda_1 = 4.6760 \text{Å} \tag{2}$$

$$\frac{\lambda_2}{2} = \frac{a}{\sqrt{\vec{h_2}^2 + \vec{k_2}^2 + \vec{l_2}^2}} = \frac{4.0495 \cdot 10^{-9} m}{\sqrt{4+0+0}}; \qquad \lambda_2 = 4.0495 \text{Å} \tag{3}$$

The timing measurements at the maximum of these peaks correspond to neutrons of known energies. We can calculate path length from wavelength, now:

$$L_i = \frac{h(t_i - T_E)}{m_n \lambda_i} \tag{4}$$

Evaluating the time of flight still requires knowledge of the absolute "emission time" from the moderator. We could try to estimate this by correcting the T0, again using measured distances, but it would be more accurate to solve for it from our two scattering peaks. Since both path length solutions refer to the same emission time T_E, we can eliminate this variable:

$$L = \frac{h}{m_n} \frac{(t_i - t_j)}{(\lambda_i - \lambda_j)} \tag{5}$$

Since we already know the wavelengths, we can find the path length by making a precise measurement of the *relative* arrival time of the neutrons. To precisely measure the time between the arrivals, we can isolate the bragg peaks:

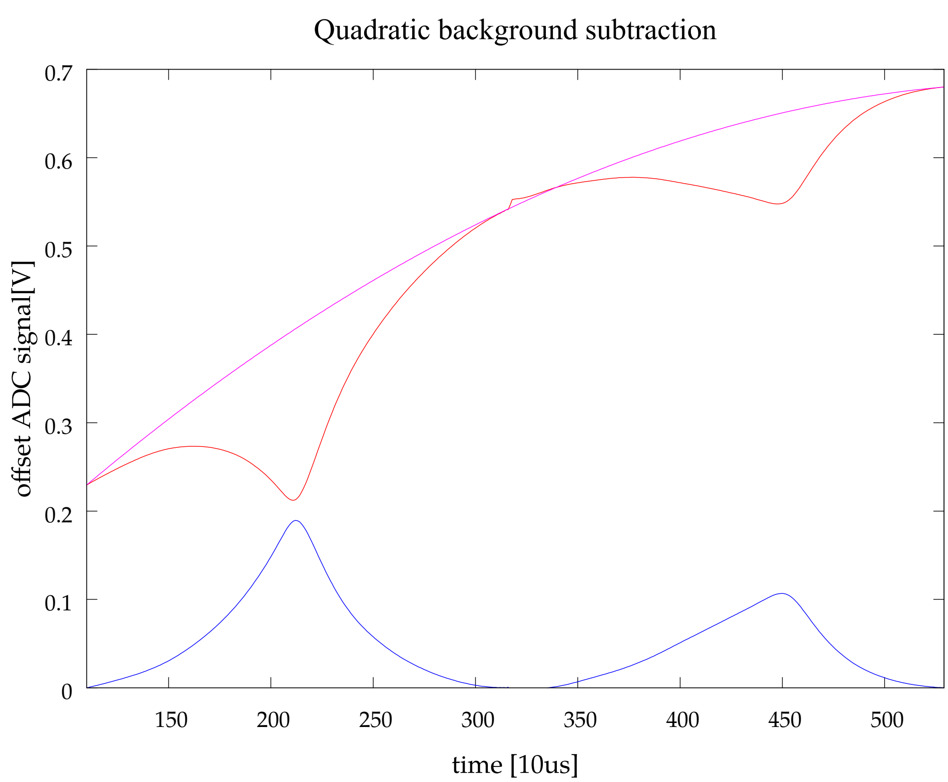


Figure 2: Isolating the scattering peaks.

Now we can calculate the path length to M1:

$$L = \frac{h}{m_n} \frac{(t_i - t_j)}{(\lambda_i - \lambda_j)} = \frac{h}{m_n} \frac{2.39411 \cdot 10^{-3} s}{6.264598 \cdot 10^{-11} m} = 15.119m \tag{6}$$

and the uncertainty (using half of the DAQ resolution, or 5μs):

$$\sigma_L = \frac{h}{m_n} \frac{\sqrt{\sigma_{t_1}^2 + \sigma_{t_2}^2}}{(\lambda_i - \lambda_j)} = \frac{h}{m_n} \frac{7.07106 \cdot 10^{-6} s}{6.264598 \cdot 10^{-11} m} = 0.045m \tag{7}$$

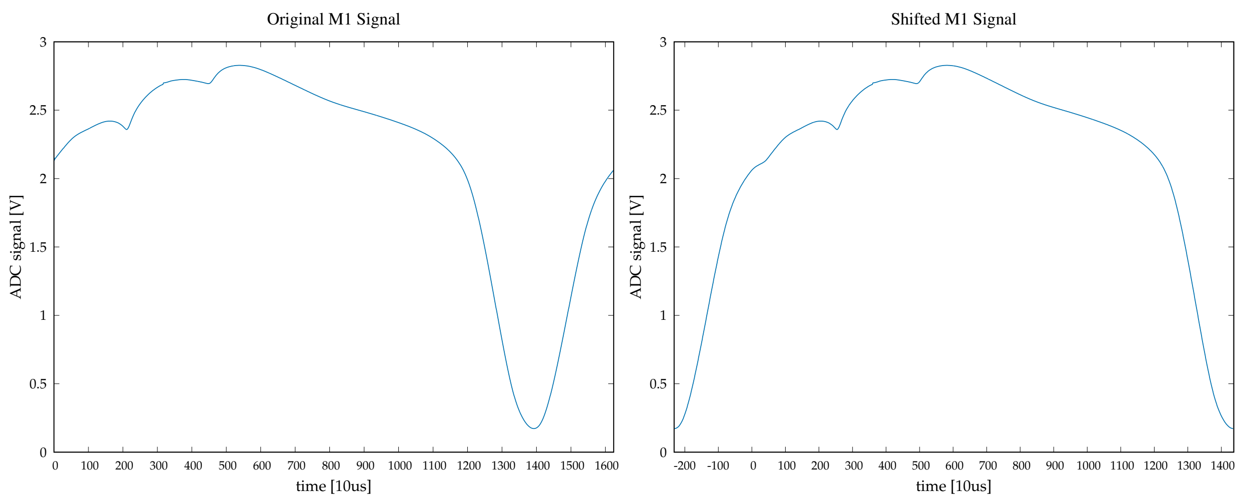
So we can calculate the path length to the center of the first beam monitor using the two Al edges in the spectrum with a useful precision.

3 Time of Flight

The information contained in the locations of the bragg peaks can also give an estimate of the absolute emission time of the neutrons. Since $L_i = L_j$, we can calculate T_E:

$$T_E = \frac{\lambda_i t_j - \lambda_j t_i}{\lambda_j - \lambda_i} = 13.424ms \tag{8}$$

This determines the time of flight to the zeroth timebin of the monitor signal. The recording timings are calibrated to measure a centered pulse in the first columnn of cells of the detector, which is downstream from the monitor. We would like to view an ordered velocity spectrum, so we have to shift our signal by -2.74ms to see the true time zero:



Our true time zero is 10.504ms. Now that we have the absolute distance traveled, and the absolute time, we can construct $E(t)$ from $v(t)$.

There is a further correction that must be made before we can create our energy distribution. The beam monitor M1 uses ^{3}He to detect neutrons. The neutron cross section of ^{3}He has a wavelength dependence of the form $\sigma[\lambda] \propto \lambda$, meaning that our observed intensity $S(\lambda)$ is proportional to the true intensity $I(\lambda)$:

$$I(\lambda) \propto S(\lambda) \cdot \lambda$$

Since we are interested in building a relative probablity distribution, we are only concerned with the relative sensitivy of the monitor to different wavelengths. So if we scale the measured intensity in the monitor by $\frac{1}{\lambda}$ before normalization, we will produce the correct distribution.

Finally, we have the desired spectrum:

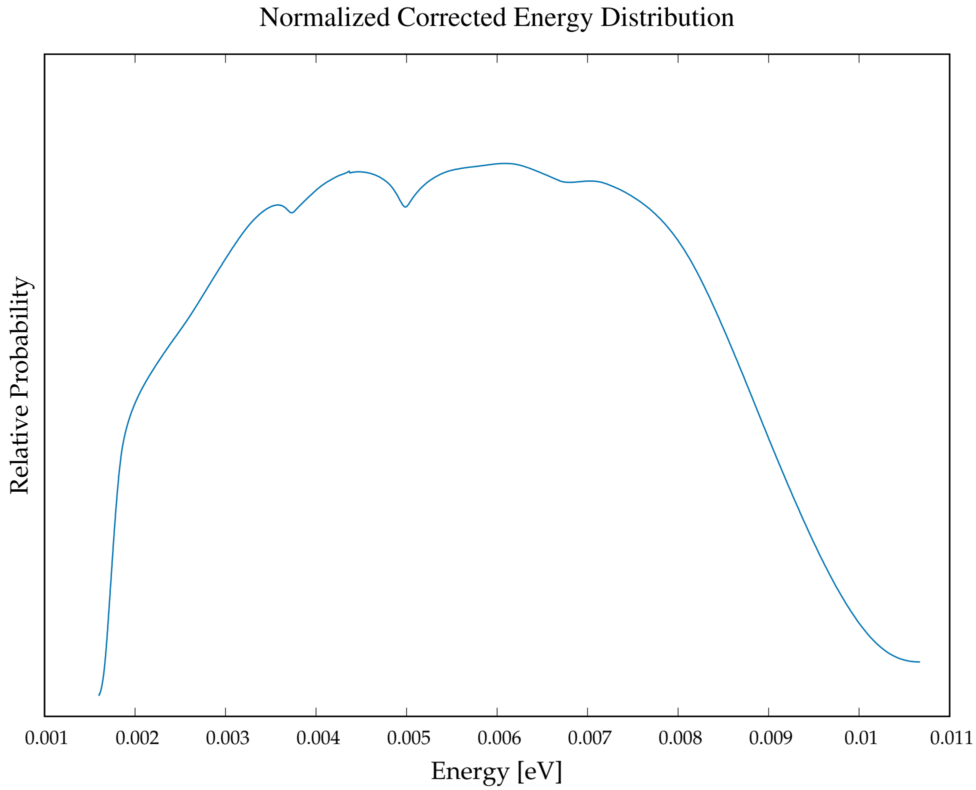


Figure 4: Distribution of neutron energies at M1.

A Emission time

We would also like to check how good our assumption is about the relationship between velocity and energy. There is no direct measurement of this we could use in our calculation, but simulations of the emission of neutrons from the moderator are available from the SNS Neutronics group. For our region of interest, the neutron brightness at the beginning of the flight path looks like this:

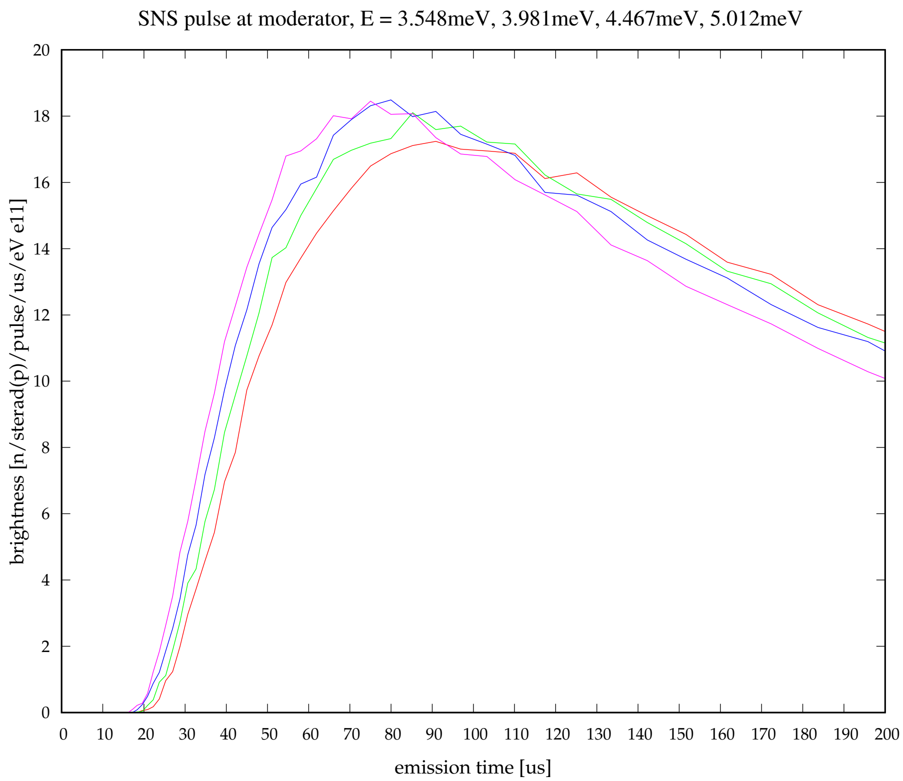


Figure 5: Pulse shape, at the moderator, at different energies.

This simulation suggests that the above calculation will overestimate the difference in the times of flight of the two selected neutron energies, because the higher energy neutrons leave first. This error will be on the order of 10μs. So, if the effect could be measured, the correction to our path length calculation would be -.06m, which is comparable to the existing uncertainty in the measurement.

B BL-13 Design Document

We would like to compare our calculated distance with the physical distance to see how close the values are. The intended path length for beamline 13 is specified in the design document "FUND13NPDG00M8U8713A001." These drawings contain the distances to the end of the guide flange, where M1 is located. Based on this geometry, and the known offset of M1 from the guide flange, we can get an estimate of the ideal direct path length,

$$L_d = \frac{L_g - L_i}{\cos\theta_d} + L_i + L_o = 598.471in = 15.201m, \tag{9}$$

where L_g is the distance to the guide flange, L_i is the distance to the intersection point, θ is the deflection angle, and L_o is how far the monitor is from the edge of the flange. This differs from our above calculation by 8cm.

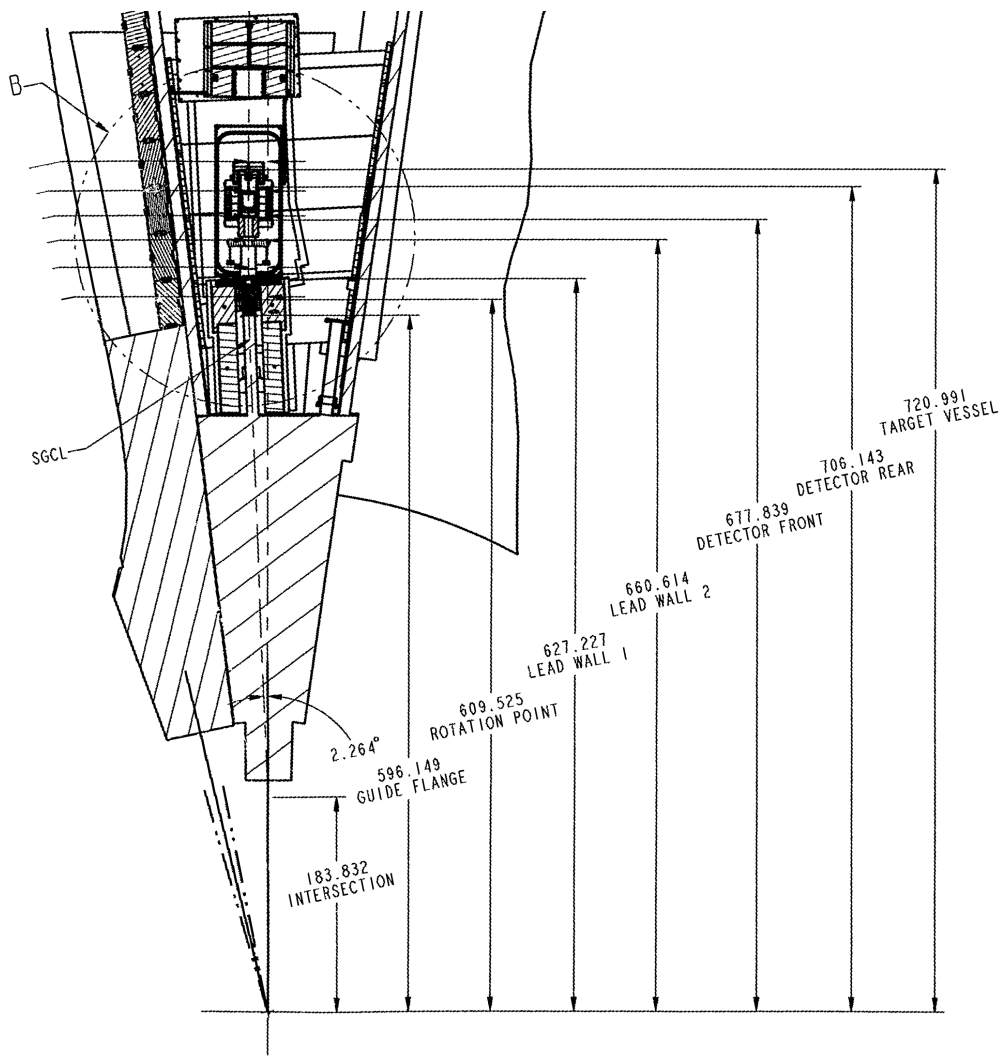


Figure 6: B-13 Design Document.