Thermal averaged n-^{3}He cross section

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The zero-temperature Laboratory cross section, σ, for projectile A on target B is defined by considering flux removal from a beam of particles of type A on a target of stationary particles of type B.

$$\frac{d\Phi_A}{dz} = -N_B\,\sigma\,\Phi_A$$

Φ_A is the flux of projectiles.

N_B is the number of target particles per unit area.

σ is the cross section.

We want to describe the situation where a neutron (projectile) with velocity v in the Lab frame, passes through a gas of ^{3}He (target) at a temperature T. For the n-^{3}He experiment the thermal velocity of the ^{3}He atoms is larger than the velocity of the neutron beam. We must define what is meant by the "thermal average cross section". In the Lab. frame, the cross section for neutrons interaction with stationary ^{3}He atoms is inversely proportional to the neutron velocity.

$$\sigma = \frac{\sigma_1 v_1}{v}$$

v_1 is a reference velocity and σ_1 is a reference cross section. How can we say that the thermal average neutron cross section is inversely proportional to the neutron velocity?

Proceed by asking the question, "What is the time rate of change of the density of neutrons that have not interacted in the neutron rest frame?" Transform to the system where the neutron rest frame. Assume that In the Lab frame the velocity distribution of ^{3}He atoms is

$$P\left(w_x, w_y, w_z\right).$$

In the neutron rest frame the distribution of ^{3}He velocities is

$$P\left(w_x, w_y, w_z - v\right).$$

For each value of the ^{3}He velocity, the cross section is well defined and is equal to the Laboratory cross section for neutrons on ^{3}He at the neutron velocity $\sqrt{w_x^2 + w_y^2 + \left(w_z - v\right)^2}$. The differential rate of ^{3}He atoms having, velocity, $\vec{w} = \left(v_x, v_y, v_z - v\right)$ interacting with the stationary neutron is well defined because the neutron is at rest.

$$\frac{d^4F}{dw_x\, dw_y\, dw_z\, dt} = N_B \sigma P\left(w_x, w_y, w_z - v\right)\sqrt{w_x^2 + w_y^2 + w_z^2}$$

. The total cross section in the neutron rest frame is the same as in the Lab. frame.

$$\sigma = \frac{\sigma_1 v_1}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

The differential rate of interactions is

$$\frac{d^4F}{dw_x\, dw_y\, dw_z\, dt} = N_B \frac{\sigma_1 v_1}{\sqrt{w_x^2 + w_y^2 + w_z^2}} P\left(w_x, w_y, w_z - v\right)\sqrt{w_x^2 + w_y^2 + w_z^2} =$$

$$N_B \sigma_1 v_1 P\left(w_x, w_y, w_z - v\right)$$

The rate of removal of stationary neutrons from their population, Φ, is

$$\frac{d\Phi}{dt} = -N_B \sigma_1 v_1 \Phi .$$

Recall that $\frac{dz}{dt} = v$. Because the total cross section in the neutron rest frame is the same as in the Lab. frame.

$$\frac{d\Phi}{dz} = -\frac{N_B \sigma_1 v_1}{v}\Phi$$

This equation has the same form as that of a zero-temperature cross section and can be interpreted to mean that the thermal-average cross section of a neutron interaction with a ^{3}He gas at temperature T is inversely proportional to the neutron velocity.