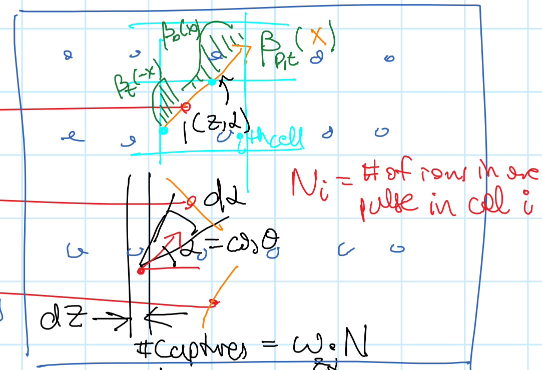
n3He simulation

Tuesday, April 7, 2015 12:18 PM

$x, y,$
v_x, v_y, v_z
foreach
MCevent

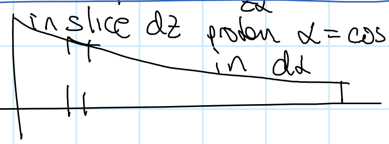
$\int \beta_p(x) + \beta_t(-x)\, dx$
$= 764\,\text{keV}$
$\overline{40\,\text{keV}}$
= total ions deposited.

$\beta_i(z,\alpha)$ = total ions deposited in cell i for one neutron decay at z, α [yield/decay]



N_i = # of ions in one pulse in cell i

captures = $\omega_{z\alpha} N$ in slice dz proton $\alpha = \cos\theta$ in $d\alpha$



$N_0 = \sum_i p_5$ returns pulse.

$N = \sum p_6$

$N_{cap} = N \sum_{\alpha z} \omega_{\alpha z} < N$

$N_i = \sum_{(x,v),(z,\alpha)} n\, \omega\, \beta_i(z,\alpha) = \sum n_i$

from McStas ntuple

$\sum_{MC\ x,v} p_5 = N_0$ $\quad i$ = batch of neutrons in a pulse w/ same x, y, v_x, v_y, v_z } MC ntuple = 1 pulse

p_5 = # of neutrons assoc. with that batch

N_0 = # of neutrons in one pulse. @ 2MW.

neutrons in one pulse

neutrons in MC event. out of SM pol.

average polarization

polarization of that event.

$$N \equiv \sum_{x,v} n,$$

$$P^2 N \equiv F \equiv \sum_{x,v} p^2 n = \langle p^2 \rangle N,$$

ions in i^{th} cell from n neutrons (MC event)

detection eff.

ions ith cell pulse

$$\beta_i N \equiv N_i \equiv \sum_{x,v,z,\alpha} n_i = \langle \beta_i(z,\alpha) \rangle N,$$

$$\alpha_i \beta_i N \equiv M_i \equiv \sum_{x,v,z,\alpha} \alpha n_i = \langle \alpha \beta_i(z,\alpha) \rangle N,$$

weight

"weighted average of $\cos\theta$"

calculating a weighted average of $\alpha = \cos\theta$

GEOMETRY FACTOR

$\text{Cov}(N_i, N_j) \quad = \sum \beta_i \beta_j \omega_{z\alpha} N$

$$\delta^2 N_{ij} = \sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} \delta^2 n_{ij} = \sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} \frac{n_i n_j}{w_{z\alpha} n} = \underbrace{\langle \beta_i \beta_j \rangle}_{\beta_{ij}} N.$$

$$n_i = \beta_i(z,\alpha)\, \overbrace{w_{z\alpha}}^{\text{capture weight.}}\, n$$

# of ions / capture event

$$N_i^{\pm} = N_i(1 \pm PA\alpha_i), \tag{11}$$

with the covariance matrix $\delta^2 N_{ij}$ after summing over both helicity states. From this we extract individual detector helicity asymmetries,

$$A_i = \frac{N_i^+ - N_i^-}{N_i^+ + N_i^-} = PA\alpha_i. \tag{12}$$

The covariance matrix of detector asymmetries A_i is

$$\delta^2 A_{ij} = \frac{\delta^2 N_{ij}}{N_i N_j} = \frac{\langle \beta_i \beta_j \rangle}{\beta_i \beta_j\, N}. \quad = \sum_{m,n,\pm\pm'} \frac{\partial A_i}{\partial N_m^{\pm}} \frac{\partial A_j}{\partial N_n^{\pm'}} \underbrace{\delta(N_m^{\pm} N_n^{\pm'})}_{\delta^2 N_{ij}} \tag{13}$$

est of phy asym   exp.

Let the extracted physics asymmetry be $A_i' = A_i/P\alpha_i$. The average extracted asymmetry over all detectors must be covariantly weighted by the matrix w:

$$w_{ij} = \delta^{-2} A_{ij}' = P^2 \alpha_i \alpha_j \delta^{-2} A_{ij}, \tag{14}$$

weight $(\sum_j w_{ij})$

$$A' \equiv \langle A_i' \rangle_w \equiv \frac{\sum_{ij} w_{ij} A_i'}{\sum_{ij} w_{ij}} = \frac{\sum_{ij} P\, \alpha_i A_j \delta^{-2} A_{ij}}{\sum_{ij} P^2 \alpha_i \alpha_j \delta^{-2} A_{ij}}. \tag{15}$$

The figure of merit equals the denominator, a measure of the effective statistics:

$$\delta^{-2} A' \equiv \sum_{ij} \delta^{-2} A_{ij}' = \sum_{ij} w_{ij} = \sum_{ij} P^2 M_i M_j \delta^{-2} N_{ij} \equiv \alpha P^2 N, \tag{16}$$

$$\alpha = \sum_{ij} \frac{\alpha_i}{\beta_i} \frac{\alpha_j}{\beta_j} \langle \beta_{ij} \rangle$$

$$\delta \bar{x} \sim \frac{1}{\sqrt{N}} \qquad N \sim \frac{1}{(\delta x)^2} \sim \delta^{-2} x$$

$$\omega \sim N \sim \delta^{-2} x$$

$$A_{kk^*} = \frac{(N_k^+ - N_{k^*}^-) - (N_k^- - N_{k^*}^+)}{+}$$

$$= \frac{A_k N_k - A_{k^*} \cdot N_{k^*}}{N_k + N_{k^*}}$$

$$\delta^2\left(\sum_i \alpha_i x_i\right) = \sum_{mn} \frac{\partial \sum_i \alpha_i x_i}{\partial x_m} \frac{\partial \sum_j \alpha_j x_j}{\partial x_n} \delta^2 x_{mn}$$

$$= w_k A_k + w_{k^*} A_{k^*}$$

$$= \sum_{mn} \alpha_m \alpha_n \delta^2 x_{mn}$$

$$\delta^2 A_{kk^*, \ell\ell^*} = \begin{pmatrix} w_k & w_{k^*} \end{pmatrix} \begin{pmatrix} \delta^2 A_{k\ell} & \delta^2 A_{k^*\ell} \\ \delta^2 A_{k\ell^*} & \delta^2 A_{k^*\ell^*} \end{pmatrix} \begin{pmatrix} w_\ell \\ w_{\ell^*} \end{pmatrix}$$

$$w_k = \frac{N_k}{N_k + N_{k^*}} \qquad w_{k^*} = \frac{N_{k^*}}{N_k + N_{k^*}}$$

if $w_k = w_{k^*}$ $\qquad w_k + w_{k^*} = 1$

$A_{k\ell} = A_{k^*\ell} = A_{k\ell^*} = A_{k^*\ell^*}$

$$\delta^2 A_{kk^*\,\ell\ell^*} = \left[(w_k + w_{k^*})\,\delta^2 A_{k\ell} \quad (w_k + w_{k^*})\,\delta^2 A_\ell \right] \begin{bmatrix} w_\ell \\ w_{\ell^*} \end{bmatrix}$$

$$= \delta^2 A_{k\ell}$$