Beam Off Asymmetry Analysis - Covariance Weighted Averaging

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November 10, 2017

Beam Off Asymmetry Calculation

The single wire instrumental asymmetries, $A_{i,j,t,q}$, were calculated using a simple difference formula normalized by one volt to render it unitless.

$$A_{i,j,t,q} = \frac{Y_{i,j,t,k=\mathrm{even}} - Y_{i,j,t,k=\mathrm{odd}}}{1V} \tag{1}$$

where i is the wire number, j is the run number, t is the time bin number, k is the pulse number, and q is the asymmetry number. Pulse and asymmetry numbers are indexed starting at zero.
Note: Beam on physics asymmetries were calculated over time bins $5-44$, and that time bin range was used for all parts of the following analysis.

Covariance and Related Formulas

$$\text{Mean of X: } \bar{X} = \frac{1}{N}\sum_{i}^{N} X_i \tag{2}$$

$$\text{Covariance: } Cov(X,Y) = \frac{1}{N}\sum_{i=0}^{N}(X_i - \bar{X})(Y_i - \bar{Y}) \tag{3}$$

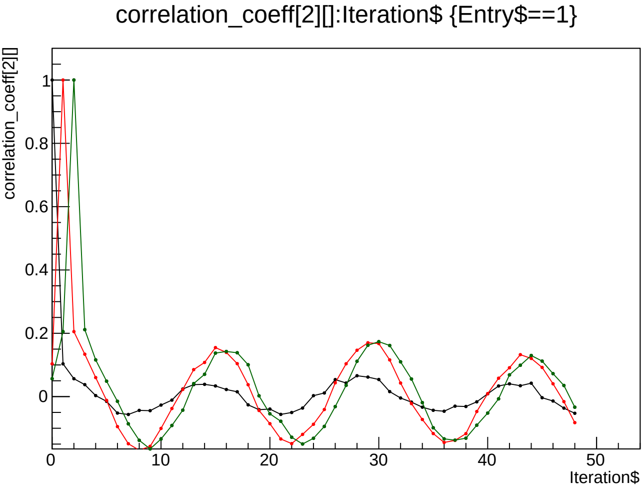
$$= \overline{XY} - \bar{X}\bar{Y}(\text{not accurate for floats}) \tag{4}$$

$$\text{Variance: } Cov(X,X) = \frac{1}{N}\sum_{i=0}^{N}(X_i - \bar{X})(X_i - \bar{X}) \tag{5}$$

$$\text{Std. Deviation: } \sigma_X = \sqrt{Cov(X,X)} \tag{6}$$

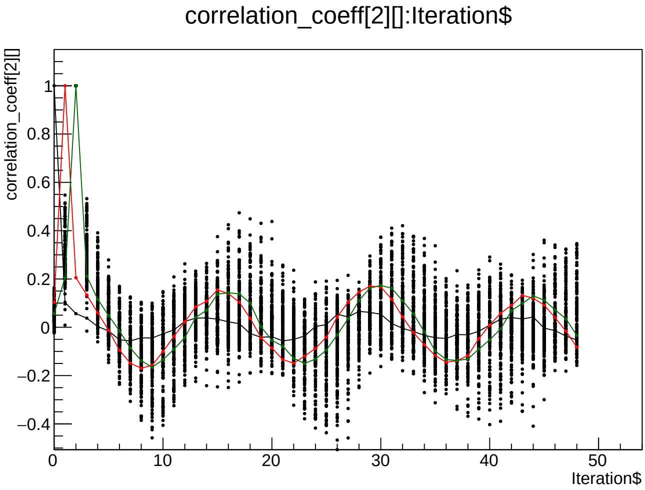
$$\text{Correlation Coef. } \rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \tag{7}$$

Correlations for time bins 0,1,2 wire 0 run 17785



Correlations for time bins 0,1,2 run 17785 and all wires

time bin 0



Covariance Weighted Mean and Uncertainty

The error weighted mean is:

$$\bar{A}_i = \frac{\sum_t A_{i,t}/Cov(A_{i,t}, A_{i,t})}{\sum_t 1/Cov(A_{i,t}, A_{i,t})} = \frac{\sum_t A_{i,t}/\sigma^2_{A_{i,t}}}{\sum_t 1/\sigma^2_{A_{i,t}}} \tag{8}$$

$$\sigma_{\bar{A}_i} = \frac{1}{\sqrt{\sum_t 1/\sigma^2_{A_{i,t}}}} \tag{9}$$

The covariance weighted mean is:

$$\bar{A}_i = \frac{\sum_t A_{i,t}/\left(\sum_{t'} Cov(A_{i,t}, A_{i,t'})\right)}{\sum_t 1/\left(\sum_{t'} Cov(A_{i,t}, A_{i,t'})\right)} \tag{10}$$

$$\sigma_{\bar{A}_i} = \frac{1}{\sqrt{\sum_t \frac{1}{\left(\sum_{t'} Cov(A_{i,t}, A_{i,t'})\right)}}} \tag{11}$$

Old Method

1. Calculate Asymmetry
$$A_{i,j,t,q} = \frac{Y_{i,j,t,k=\text{even}} - Y_{i,j,t,k=\text{odd}}}{1V} \tag{12}$$
2. Accumulate and store sum of asymmetry and sum of square of asymmetry for each time bin of each wire for each run in range in a TTree along with number of entries from each run.
3. Sum the sum and sum of squares for each time bin of each wire over all runs.
4. Calculate mean and standard error for the values for each wire and time bin.
5. Perform error weighted average of the time bin averages and standard errors.
6. Perform weighted sums of each wire value using physics asymmetry uncertainties as shown two slides previous in the slide from Kabir's presentation.

The goal of this is to not integrate out time effects over pulses. While the mean does not change with the different time averaging the standard error does.

Covariance Matrix Calculation Method

1. The covariance matrix, M, is a $N \times N$ matrix the gives the variance and covariance of N values.
2. The matrix is symmetric, $M_{ij} = M_{ji}$
3. The covariance matrix was calculated for all time bins for each wire of **ONE** run.
4. This was used to set the weights for the covariance weighted sum of the mean time bin asymmetries calculated over all runs in step number 5 on the previous slide.

Issues:

- When recreating the error weighted average from the covariance weighting numbers the errors are $\sqrt{570}$ larger due to the fewer number of runs used.
- for approximately half the wires when calculating $\sigma_{\bar{A}_i}$ the $\sum_t \frac{1}{\left(\sum_{t'} Cov(A_{i,t}, A_{i,t'})\right)}$ is negative preventing square root from being calculated.