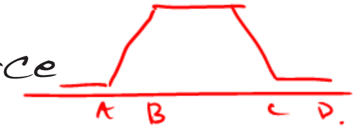
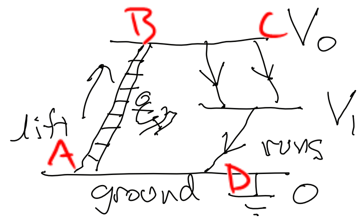
Section 7.1 - Electromotive Force



* review

~ current element $\quad q \leftrightarrow \lambda \leftrightarrow \sigma \leftrightarrow \rho$

$\qquad\qquad\qquad\qquad\quad q\vec{v} \leftrightarrow I \leftrightarrow K \leftrightarrow J$

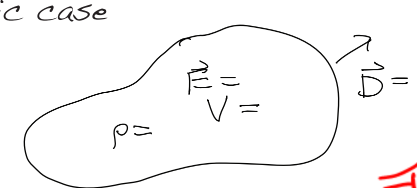


~ continuity $\frac{\partial}{\partial t}\rho + \nabla\cdot J = 0$ conservation of charge

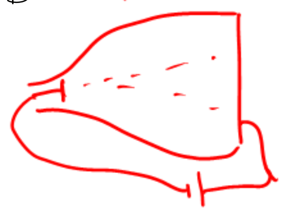
~ potential $\vec{E} = -\nabla V$ conservation of energy

* conductors

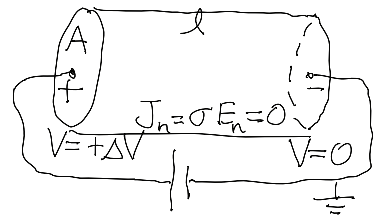
~ static case



ΔV



~ steady current RESISTOR



$I = \Phi_J = JA$

$\Delta V = \mathcal{E}_E = E\cdot d$

~ what if $\Delta V \neq 0$?

$m\vec{a} = \vec{F} = q\vec{E}$

~ current $\boxed{\vec{J} = \sigma\vec{E}}$ $\boxed{D = \varepsilon E}$ $\boxed{B = \mu H}$

(third constitutive equation)

conductivity

ρ = resistivity $= \frac{1}{\sigma}$

~ resistor vs. CRT

$b\vec{v}_d = -\vec{F}_f = \vec{F}_e = q\vec{E}$

free charge density $\quad \vec{J} = \rho_f\vec{v}_d = \frac{\rho_f q}{b}\vec{E}$ $\quad \frac{\rho_f q}{b} = \sigma$

~ Drude law: bumper cars

$v_d = \frac{\langle \frac{1}{2}at^2\rangle}{\langle t\rangle} = at = \frac{qE}{m}\cdot\frac{\lambda}{v_{rms}}$

$b = \frac{m v_{rms}}{\lambda} \qquad \sigma = \frac{(\rho_f q)}{b} = \frac{(nfq^2)\lambda}{m\, v_{rms}}$ MFP.

t = time between collisions

λ = mean free path

nf = atomic density × # carriers/atom

$\nabla^2 V = 0 \qquad$ B.C.'s ?

sol'n: $V = \Delta V \cdot z/\ell$

$I = \vec{J}\cdot\vec{A} = \sigma E A = \sigma A \frac{\Delta V}{\ell}$

$= \frac{\Delta V}{R} \qquad R = \frac{\rho\ell}{A} = \frac{\ell}{\sigma A} \qquad \rho \equiv$ resistivity, $\sigma \equiv$ conductivity

$\frac{\Delta V}{I} = R = \frac{\mathcal{E}_E}{\Phi_J} = \frac{\ell}{A\sigma}$

$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$

G = conductance $= \frac{1}{R} = \frac{\sigma A}{\ell}$

~ versus CAPACITOR

flux of D $\qquad$ flow E

$Q = C\,\Delta V \qquad \boxed{C = \frac{\varepsilon A}{\ell}}$

$U = \frac{1}{2}Q\Delta V = \frac{1}{2}\frac{Q^2}{\Delta V} = \frac{1}{2}C\Delta V^2$

* power dissipation

$P = \vec{F}\cdot\vec{v}_d = q\vec{E}\cdot\vec{v}_d \qquad u = \frac{du}{dt} = \frac{\Delta P}{\Delta\tau} = \rho_f v_d\cdot E = \vec{J}\cdot\vec{E} = \sigma E^2 = \rho J^2$

$u = \frac{\Delta W}{\Delta\tau} = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\varepsilon E^2$

* relaxation time

$\frac{\partial\rho_f}{\partial t} = \nabla\cdot\vec{J} = \frac{\sigma}{\varepsilon}\nabla\cdot\vec{D} = \frac{\sigma}{\varepsilon}\rho_f(t) \qquad \rho = \rho_0 e^{-\sigma/\varepsilon\, t} \qquad \tau = \frac{\varepsilon}{\sigma} = RC$

$\tau = \frac{\varepsilon}{\sigma} = \frac{1/376.7\ c\cdot\Omega}{1/1.678\ \mu\Omega\cdot cm} = 0.445\ \text{Å}/c = 1.45\times10^{-19}\,s$

* electromotive force (emf)

~ electromotance more correct! compare: magnetomotance (H/W4, #3)

~ forces on electrons from E and other sources (chemical, B, ...)

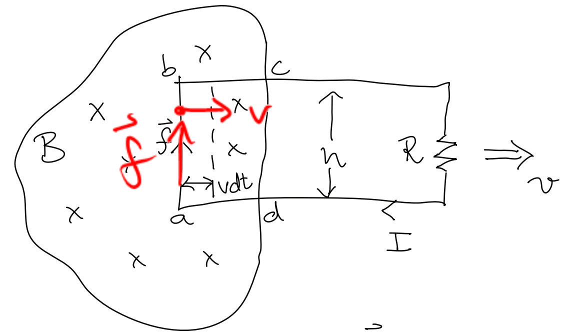
~ not quite $\mathcal{E}_E = \int \vec{E}\cdot d\vec{l}$ since $\mathcal{E}_E = 0 = \int \vec{f} + \vec{E}\, dl$

$\vec{F} = q\vec{f}$ generalization of $\vec{E}$ $\qquad \mathcal{E} = \int \vec{f}\cdot d\vec{l}$ (emf)

* motional emf - magnetic forces

$$F = q(\vec{E} + v\times\vec{B}) \qquad \vec{f} = \vec{v}\times\vec{B}$$

$$\mathcal{E} = \oint \vec{f}_{mag}\cdot d\vec{l} = vBh$$

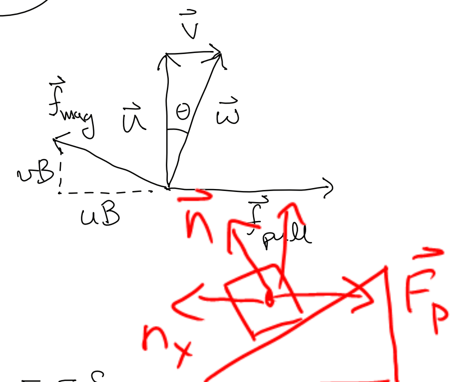


~ relation to flux: precursor to Faraday's law

$$\Phi_B = \int \vec{B}\cdot d\vec{a} = Bhx$$

$$\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv = -\mathcal{E}$$

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$



~ conservation of energy: magnetic force does no work!

$$\int \vec{f}_{pull}\cdot d\vec{l} = uB\frac{h}{\cos\theta}\sin\theta = uB\cdot h\cdot\frac{w}{u}\cdot\frac{v}{w} = Bhv = -\mathcal{E}$$

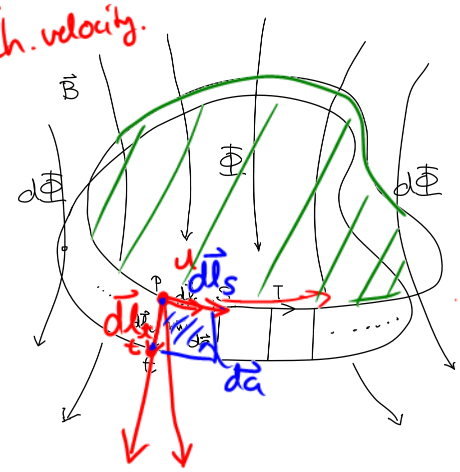
~ general proof

drift vel.

mech. velocity

$$\vec{w} = \vec{u} + \vec{v}$$
$$d\vec{l}_s = \vec{u}\,ds$$
$$d\vec{l}_t = \vec{v}\,dt$$

$$\mathcal{E} = \oint \vec{f}_{mag}\cdot d\vec{l}_s = \oint \vec{w}\times\vec{B}\cdot d\vec{l}_s$$
$$= -\oint \vec{B}\cdot(\vec{v}+\vec{u})\times d\vec{l}_s = \oint \vec{B}\cdot\frac{d\vec{l}_t\times d\vec{l}_s}{dt}$$
$$= \oint \vec{B}\cdot\frac{d\vec{a}}{dt} = -\frac{d\Phi}{dt}$$



~ what about 'work' done by electromagnet lifting a car in the junkyard?