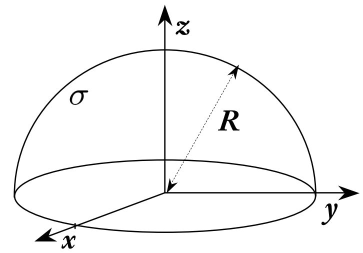
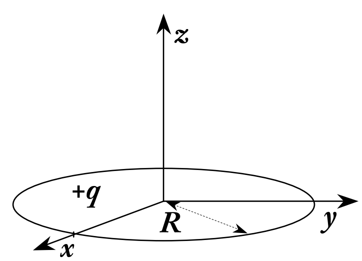
University of Kentucky, Physics 416G
FINAL EXAM, 2012-12-10 8:00-10:00 AM

Instructions: The exam is closed book and timed (2 hours). Only 5 pages will be graded—cross out one page before turning it in. You may refer to a 1-page formula sheet (no proofs or examples). Show all steps of your calculations. Pace yourself. May the Flux be with you, and may you achieve your full Potential (and not get grounded). [95 pts total]

[8 pts] 1. a) Find the magnitude and direction of the electric field at the center of curvature, $\boldsymbol{r} = (0, 0, 0)$ of a hemispherical bowl of radius R and constant surface charge density σ.



[7 pts] b. Calculate the potential $V(\boldsymbol{r})$ at the point $\boldsymbol{r} = (0, 0, z)$ on the z-axis from a uniformly distributed ring of charge q of radius R in the xy-plane, centered about the origin.

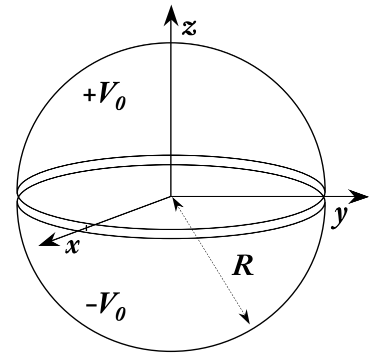


[6 pts] 2. a) Given $V(r,\theta,\phi) = q/4\pi\epsilon_0 r$, calculate $\boldsymbol{E}$ and show that $\rho(\boldsymbol{r}) = q\,\delta^3(\boldsymbol{r})$.

[10 pts] b) Derive Coulomb's law $\boldsymbol{E} = \int đq'\,\hat{\boldsymbol{\mathscr{r}}}/4\pi\epsilon_0 \boldsymbol{\mathscr{r}}^2$, from the first one and a half Maxwell equations: $\boldsymbol{\nabla}\cdot\boldsymbol{E} = \rho/\epsilon_0$ AND $\boldsymbol{\nabla}\times\boldsymbol{E} = 0$.

[4 pts] c) Derive the boundary condition $D_{2n} - D_{1n} = \sigma_f$ from Gauss' law and use it to show that $-\epsilon_2 \frac{\partial V_2}{\partial n} + \epsilon_1 \frac{\partial V_1}{\partial n} = \sigma_f$ on the boundary.

3. A spherical capacitor of two hemispherical conducting shells is filled with peanut butter, with dielectric constant ϵ_r=3.22. The top shell is held at potential $+V_0$ and the bottom shell at $-V_0$. At a cost of [-3 pts] you can ignore the peanut butter.



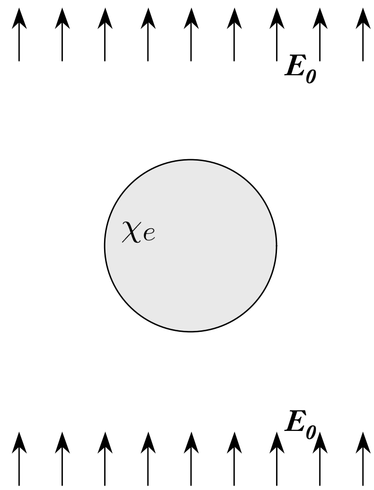
[3 pts] a) Draw field lines and equipotentials on the inside and outside.

[10 pts] b) Solve the boundary value problem to calculate the potential everywhere INSIDE of the sphere (only the FIRST nonzero term).

[5 pts] c) Calculate the total free charge on the INSIDE of the top hemispherical shell.

[2 pts] d) Calculate the partial capacitance (ignoring the outside for simplicity).

4. A uniform dielectric CYLINDER along the z-axis of radius R and susceptibility χ_e is placed in a uniform external electric field $\boldsymbol{E} = E_0\hat{\boldsymbol{y}}$ [so you can ignore $\cos(m\phi)$ terms].



[12 pts] a) Calculate the potential inside and out as a dielectric boundary value problem (using ϵ, not σ_b).

[5 pts] b) Calculate the bound charge surface and volume densitites σ_b and ρ_b.

[3 pts] c) Draw the field lines and equipotentials in the figure.

5. [10 pts] a) Compare and contrast properties of electric flux $\Phi_E = \int \boldsymbol{E}\cdot d\boldsymbol{a}$, and flow $\mathcal{E}_E = \int \boldsymbol{E}\cdot d\boldsymbol{l}$. Describe their relation to each other, the electric field $\boldsymbol{E}$, sources, and to electrostatic concepts discussed this semester. How are they visualized geometrically? Discuss the flux of bound and free charge. Be as complete as possible.

b) [10] Discuss five key mathematical theorems or principles and their relation to electrostatics.

6. [10 pts] Calculate the electric monopole $Q^{(0)}$, dipole $Q^{(1)}$, and quadrupole $Q^{(2)}$ moments of a sphere of radius R about the origin with the surface charge distribution $\sigma(\theta, \phi) = \sigma_0 \sin^2\theta$. [The ℓ^{th} multipole moment is $Q^{(\ell)} = \int đq' r^\ell P_\ell(\cos\theta)$.]