Section 4.3 - Electric Displacement $\vec{D}$

* reviews: parallels between E and P
- ~ what are the units of $\varepsilon_0\vec{E}$? $\vec{P}$?
- ~ both are vector fields (functions of position)
- ~ the field lines (flux) are associated with charge (Dr. Jekyll or Mr. Hyde ??)
- ~ the two fields are related: E induces P in a dielectric

$$\Phi_{\varepsilon_0 E} = Q \qquad \nabla\cdot\varepsilon_0\vec{E} = \rho \qquad \hat{n}\cdot\Delta\varepsilon_0\vec{E} = \sigma \qquad \text{total charge}$$

$$(+)\ \Phi_P = -Q_b \qquad (+)\ \nabla\cdot\vec{P} = -\rho_b \qquad (+)\ \hat{n}\cdot\Delta\vec{P} = -\sigma_b \qquad -\ \text{bound charge}$$

$$\Phi_D = Q_f \qquad \nabla\cdot\vec{D} = \rho_f \qquad \hat{n}\cdot\Delta\vec{D} = \sigma_f \qquad = \text{free charge}$$

$$D_2^{\perp} - D_1^{\perp} = \sigma_f$$

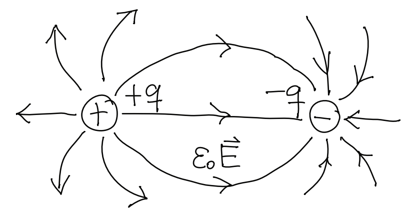
* new field: D = "electric displacement"
- ~ defined by the "constitutive equation": $\boxed{\vec{D} = \varepsilon_0\vec{E} + \vec{P}}$
- ~ associated with the free charge: lines of D flux go from (+) to (−) free charge
- ~ iterative cycle:
  - a) free charge generates E
  - b) E causes P, diplaced bound charge ρ_b σ_b
  - c) the field from bound charge modifies E
- ~ direct calculation procedure with D
  - a) calculate D directly from free charge only
  - b) obtain P from D using consititutive relation
  - c) the electric field is: $\varepsilon_0\vec{E} = \vec{D} - \vec{P}$

* differences between $\varepsilon_0 E$, P, and D:
- ~ equipotentials associated with force $\vec{F} = q\vec{E}$ only for the electric field
- ~ ρ generates E, but P induces ρ_b
- ~ $\mathcal{E}_{\Gamma} = 0 \qquad \nabla\times\vec{E} = \vec{0} \qquad \hat{n}\times\Delta\vec{E} = \vec{0}$
- ~ $\vec{E} = \int\frac{dq\,\hat{\mathscr{r}}}{4\pi\varepsilon_0\mathscr{r}^2} \qquad \vec{E}_b = \int\frac{dq_b\,\hat{\mathscr{r}}}{4\pi\varepsilon_0\mathscr{r}^2} \qquad \vec{E}_f = \int\frac{dq_f\,\hat{\mathscr{r}}}{4\pi\varepsilon_0\mathscr{r}^2}$

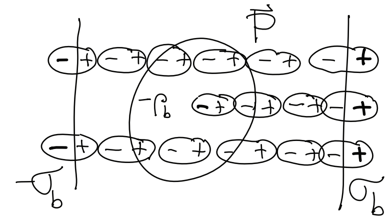
  note: not P or D in these formulas!

* you need both $\nabla\cdot\vec{D} = \rho_f$ and $\nabla\times\vec{E} = 0$ to solve!

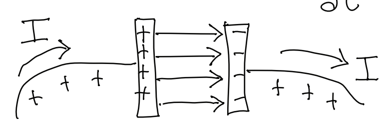
Electric field "E"



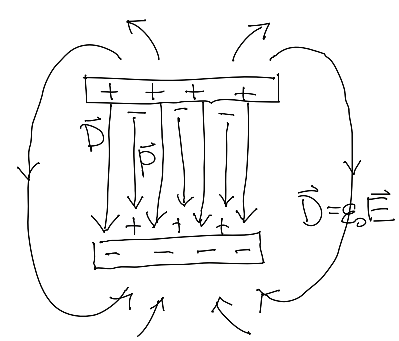
Polarization "P"



"Displacement current" (Maxwell) $\vec{J}_d = \frac{\partial\vec{D}}{\partial t}$



$$I_d = \int\vec{J}_d\cdot d\vec{a} = \int\frac{\partial\vec{D}}{\partial t}\cdot d\vec{a} = \frac{\partial\Phi_D}{\partial t}$$



$\varepsilon_0 E = D - P$ inside

$\varepsilon_0 E = D$ outside