

Section 4.4.1 - Linear Dielectrics

* going from polarizability (α) to susceptibility (χ_e)

~ atoms: $\vec{p} = \alpha \vec{E}$

~ material: $\vec{P} = \epsilon_0 \chi_e \vec{E} = \frac{\Delta \vec{P}}{\Delta \vec{E}} = \frac{\Delta \vec{P}}{\Delta \vec{E}} \vec{P} = N \alpha \vec{E}$

$\epsilon_0 \chi_e \equiv N \alpha$ (See HW9 for refinements)

* material properties:

~ linear: χ_e is independent of field magnitude $|\vec{E}|$

~ isotropic: χ_e is a scalar (independent of direction $\hat{E}$)

~ homogeneous: χ_e is independent of position

~ nonisotropic material:

$\vec{P} = \epsilon_0 \tilde{\chi}_e \vec{E}$ (like $\tilde{\alpha}$)

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

* permeability: absolute $\epsilon = \epsilon_0 \epsilon_r$, relative $\epsilon_r = K$ (dielectric const.)

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 (1 + \chi_e) \vec{E} = \epsilon_0 \epsilon_r \vec{E} = \epsilon \vec{E}$

~ constitutive eq: $\vec{D} = \epsilon \vec{E}$

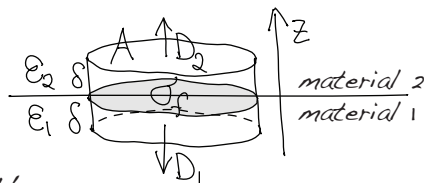
~ property of the material: $\epsilon_r = 1 + \chi_e = \epsilon / \epsilon_0$

others: $\vec{B} = \mu \vec{H}$ $\vec{J} = \sigma \vec{E}$

* continuity boundary conditions

a) FLUX

$\nabla \cdot \vec{D} = \rho_f$



~ Gaussian pillbox

$\Phi_D = \hat{z} \cdot \vec{D}_2 A - \hat{z} \cdot \vec{D}_1 A = \sigma_f A = Q_f$

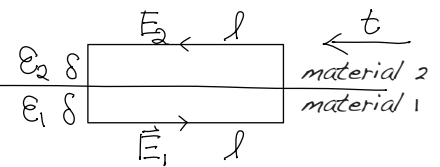
~ Integration of $\nabla \cdot \vec{D} = \rho_f$ across boundary

$\lim_{\delta \rightarrow 0} \int_{-\delta}^{\delta} dz \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right) = \int_{-\delta}^{\delta} \sigma_f \delta(z-z') dz$

$\int_{-\delta}^{\delta} dD_z = \hat{n} \cdot \Delta \vec{D} = \sigma_f$ $-\epsilon_2 \frac{\partial V_2}{\partial n} + \epsilon_1 \frac{\partial V_1}{\partial n} = \sigma_f$

b) FLOW

$\nabla \times \vec{E} = 0$



~ Amperian loop

$\mathcal{E}_E = \hat{t} \cdot \vec{E}_2 l - \hat{t} \cdot \vec{E}_1 l = 0$

~ Integration of $\nabla \times \vec{E} = 0$ across boundary

$\lim_{\delta \rightarrow 0} \int_{-\delta}^{\delta} dz \hat{x} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \hat{y} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \hat{z} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = 0$

$= \int_{-\delta}^{\delta} \hat{x} dE_y - \hat{y} dE_x = \hat{n} \times \Delta \vec{E} = 0$ $V_2 = V_1$

~ the only difference in dielectric boundary value problems is ϵ_1, ϵ_2 in boundary cond.

* example 4.7: dielectric ball in electric field

$V_1 = \sum_{l=0}^{\infty} (A_l r^l + B_l r^{-l-1}) P_l(\cos \theta)$

$V_2 = \sum_{l=0}^{\infty} (C_l r^l + D_l r^{-l-1}) P_l(\cos \theta)$

$\lim_{r \rightarrow 0} V_1(r) \neq 0 \quad B_l = 0$

$\lim_{r \rightarrow \infty} V_2(r) = -E_0 r \cos \theta \quad C_l = -E_0 \delta_{l1}$

$V_1(R) = V_2(R) \quad A_l R^l = C_l R^l + D_l R^{-l-1}$

$-\epsilon_2 V_2'(R) + \epsilon_1 V_1'(R) = \sigma_f = 0$

$-\epsilon_2 (C_l l R^{l-1} + D_l (-l-1) R^{-l-2}) + \epsilon_1 (A_l l R^{l-1}) = 0$

if $l \neq 1 \quad D_l = A_l R^{2l+1} \quad D_l = A_l = 0$

if $l=1 \quad A_1 = -E_0 + D_1 R^{-3}$
 $-\epsilon_2 (-E_0 - 2D_1 R^{-3}) + \epsilon_1 A_1 = 0$

$-\epsilon_2 (-E_0 - 2(A_1 + E_0)) + \epsilon_1 A_1 = 0$

$3\epsilon_2 E_0 + (\epsilon_1 + 2\epsilon_2) A_1 = 0$

$A_1 = \frac{-3\epsilon_2}{\epsilon_1 + 2\epsilon_2} E_0$

if $\epsilon_1 = \epsilon_r \epsilon_2 \quad A_1 = \frac{-3}{\epsilon_r + 2} E_0$

