Sections 5.1.3, 8.1.1 – Conserved currents: continuity eq.

* Symmetries: $$\frac{d}{dt}\underbrace{\frac{\partial T}{\partial \vec{v}}}_{\vec{p}} - \underbrace{\frac{-\partial V}{\partial \vec{x}}}_{\vec{F}} = 0 \quad \text{(Lagrange)}$$

$$\frac{\partial T}{\partial \vec{v}} = \frac{d}{d\vec{v}}\tfrac{1}{2}m\vec{v}^2 = m\vec{v} = \vec{p}$$

$$\frac{d\vec{p}}{dt} = m\vec{a} = \vec{F} = -\frac{\partial V}{\partial \vec{x}}$$

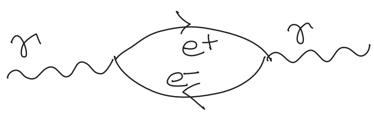
$$\text{N III}: \; F_{21} = -F_{12}$$

~ if $\mathcal{L}$ is translation invariant (symmetric)
then momentum ($\vec{p}$) is conserved in complete system

~ if laws of physics (forces) are time-invariant
then energy (E) is conserved (potential energy is stored in the force)

* Noether's thoorem: SYMMETRIES $\Longleftrightarrow$ CONSERVED CURRENTS
(the quantity is conserved, but it can move around)

~ mass?
~ charge?

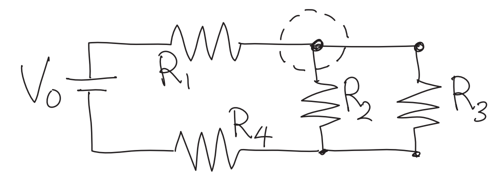


Gauge transformations $\quad \vec{E} = -\nabla V \quad$ (force independent of ground potential)

* Kirchoff's rules: conservation principles

a) loop rule: conservation of energy

$$\sum_{loop} \Delta V_i = -\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\mathcal{E}_E = -\int_S \nabla\times\vec{E}\cdot d\vec{a} = 0$$

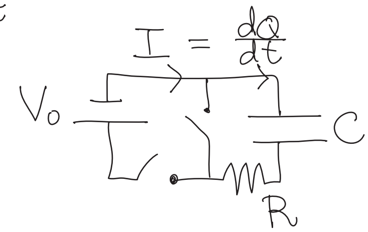


b) node rule: conservation of charge

$$\sum_{node} I_i = 0 = \oint_{\partial V} \vec{J}\cdot d\vec{a} = \int_V \nabla\cdot J \, d\tau$$

$$V_2 = V_3$$
$$I_1 = I_2 + I_3$$

~ what about a capacitor?
top plate has current coming in
but no current going out



* charge element vs. $\quad dq = q_i = \lambda\, dl = \sigma\, da = \rho\, d\tau$

current element: $\quad d\vec{q} = \vec{v}_i q_i = I\, d\vec{l} = \vec{K}\, da = \vec{J}\, d\tau$

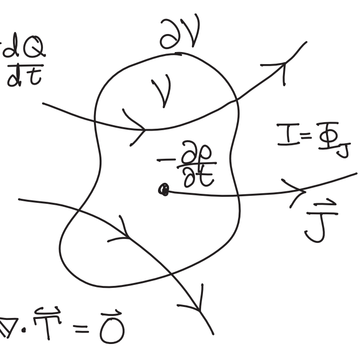
$$\vec{I} = \vec{v}\lambda = \frac{\Delta q}{\Delta t}\hat{l} \qquad \vec{K} = \vec{v}\sigma = \frac{\Delta q}{\Delta w \Delta t}\hat{l} \qquad \vec{J} = \vec{v}\rho = \frac{\Delta q}{\Delta a \Delta t}\hat{l}$$

$$I = \int \vec{K}\times d\vec{w} \qquad I = \int \vec{J}\cdot d\vec{a}$$

* continuity equation:
local conservation of charge
vs. "beam me up, Scotty"

$$I = \oint_{\partial V} \vec{J}\cdot d\vec{a} = \int_V \nabla\cdot\vec{J}\, d\tau = \int_V \frac{\partial \rho}{\partial t}\, d\tau = -\frac{dQ}{dt}$$

$$\boxed{\nabla\cdot J = -\frac{\partial \rho}{\partial t}}$$



~ 4-vector: $(c\rho, \vec{J}) = J^\mu$

$$\frac{1}{c}\frac{\partial}{\partial t} c\rho + \nabla\cdot\vec{J} \equiv \vec{\Box}\cdot\vec{J} = 0 \qquad \partial_\mu J^\mu = 0$$

~ other conserved currents: energy $\frac{\partial u}{\partial t} + \nabla\cdot\vec{S} = 0$, momentum $\frac{\partial \vec{p}}{\partial t} + \nabla\cdot\overleftrightarrow{T} = \vec{0}$

Section 7.1.1 - Conductivity: Ohm's law

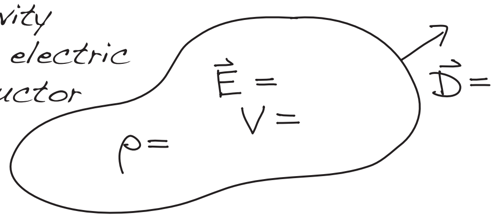
* review:

$$\begin{array}{ccccccc} q & \leftrightarrow & \lambda & \leftrightarrow & \sigma & \leftrightarrow & \rho \\ \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\ q\vec{v} & \leftrightarrow & I & \leftrightarrow & K & \leftrightarrow & J \end{array}$$

continuity: $\frac{\partial}{\partial t}\rho + \nabla \cdot J = 0$

* conductivity

~ static electric conductor



~ steady current $\Delta V \neq 0$

$$m\vec{a} = \vec{F} = q\vec{E}$$

current $\quad \vec{J} = \sigma \vec{E}$ 2nd constitutive equation

~ resistor vs cathode ray tube (CRT)?

$$b\vec{v}_d = -\vec{F}_f = \vec{F}_e = q\vec{E}$$

$$\vec{J} = \rho_f \vec{v}_d = \underbrace{\frac{\rho_f q}{b}}_{\sigma} \vec{E}$$ terminal (drift) velocity

~ Drude model: "bumper cars"

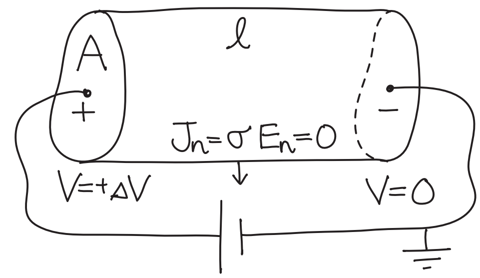
$$V_d = \frac{\langle \frac{1}{2}at^2 \rangle}{\langle t \rangle} = at = \frac{qE}{m} \cdot \frac{\lambda}{v_{rms}}$$

$$b = \frac{m v_{rms}}{\lambda} \qquad \sigma = \frac{(\rho_f q)}{b} = \frac{(nfq^2)\lambda}{m\, v_{rms}}$$

t = time between collisions
λ = mean free path
n = atomic density
f = # charge carriers/atom

* symmetries:
~ continuity equation -> Maxwell eq's
~ motional EMF => Einstein's special rel.

* RESISTOR - an electrical component



$$\nabla^2 V = 0 \qquad \text{B.C.'s} \Rightarrow V = \Delta V \cdot z/\ell$$

$$I = \vec{J} \cdot \vec{A} = \sigma E A = \sigma A/\ell \cdot \Delta V$$

$$= G\Delta V \qquad \boxed{G = \frac{\sigma A}{\ell}}$$ conductance, σ = conductivity

$$= \Delta V/R \qquad R = \frac{\rho \ell}{A}$$ resistance, ρ = resistivity

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$ power dissipated

~ vs. CAPACITOR

$$Q = C\,\Delta V \qquad \boxed{C = \frac{\varepsilon A}{\ell}}$$ capacitance, ε = permittivity

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2}\frac{Q^2}{\Delta V} = \frac{1}{2} C \Delta V^2$$

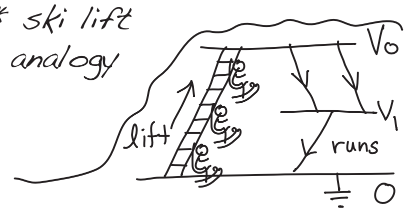
~ vs. INDUCTOR ... to be continued

* power dissipation $P = \vec{F} \cdot \vec{v}_d = q\vec{E} \cdot \vec{v}_d$

~ power density $\dot{u} = \frac{du}{dt} = \frac{\Delta P}{\Delta \tau} = \rho_f \vec{v}_d \cdot E = \vec{J} \cdot \vec{E} = \sigma E^2 = \rho J^2$

~ energy density $u = \frac{\Delta W}{\Delta \tau} = \frac{1}{2} \vec{D} \cdot \vec{E} = \frac{1}{2} \varepsilon E^2$

* ski lift analogy



* relaxation time

$$-\frac{\partial \rho_f}{\partial t} = \nabla \cdot \vec{J} = \frac{\sigma}{\varepsilon} \nabla \cdot \vec{D} = \frac{\sigma}{\varepsilon} \rho_f(t) \Rightarrow \rho = \rho_0 e^{-\sigma/\varepsilon\, t} \qquad \tau = \frac{\varepsilon}{\sigma} = RC$$

~ for copper, $\tau = \frac{\varepsilon}{\sigma} = \frac{1/376.7c\cdot\Omega}{1/1.678\,\mu\Omega\cdot cm} = 0.445\ A/C = 1.45 \times 10^{-19}\,s$