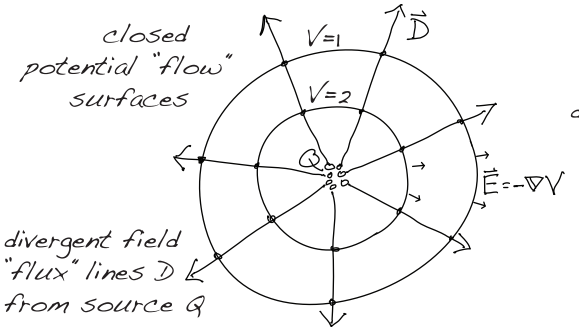
Section 5.x - Magnetic Scalar Potential

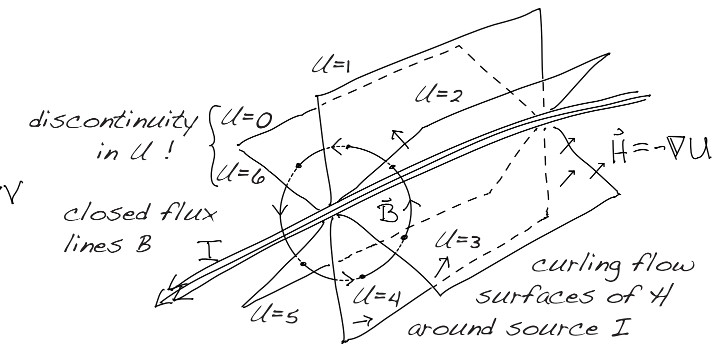
* pictorial representation of Maxwell's steady state equations

~ define $B = \mu_0 H$ to drop all the μ_0's and emphasize the "source" aspect of B

	electric	magnetic	electric	magnetic
flux:	$\nabla \cdot \vec{D} = \rho$	$\nabla \cdot \vec{B} = 0$	$\Phi_D = Q$	$\Phi_B = 0$
flow:	$\nabla \times \vec{E} = 0$	$\nabla \times \vec{H} = \vec{J}$	$\mathcal{E}_E = 0$	$\mathcal{E}_H = I$

$$U = \mathcal{E}_H = -\int_0^{\vec{r}} \vec{H} \cdot d\vec{l}$$
$$\vec{H} = -\nabla U$$





* utility of treating D, B as flux lines
and $\mathcal{E}, H$ as equipotential surfaces:

~ flux through a surface S: $\Phi_B = \int_S \vec{B} \cdot d\vec{a}$ = # of lines that poke through a surface S

~ flow along a curve/path P: $\mathcal{E}_E = \int_P \vec{E} \cdot d\vec{l}$ = # of surfaces that a path P pokes through

* scalar electric and magnetic potentials

$\vec{E} = -\nabla V$ ALWAYS $\nabla \times \vec{E} = 0$ $\qquad$ $\vec{H} = -\nabla U$ ONLY if if $\vec{J} = 0$

$\nabla \cdot \varepsilon(-\nabla V) = \rho$ Poisson's eq. $\qquad$ $\nabla \cdot \mu(-\nabla U) = 0$ Laplace's eq. ALWAYS

$V = -\frac{1}{\varepsilon} \nabla^{-2} \rho = \frac{-1}{\varepsilon} \int \frac{d\tau' \rho}{\mathscr{r}} + \nabla^{-2} 0$ $\qquad$ ~ solve $\nabla^2 U = 0$ with appropriate B.C.'s

* discontinuities in U

a) at I: the edge of each H sheet is an I line

b) around I: the U=0,6 surfaces coincide - a 'branch cut' on U extends from each I line

~ U is well defined in a simply connected region or one that does not link any current

* boundary conditions

$\nabla \to \Delta \hat{n}$ $\quad \rho \to \sigma$, $\vec{J} \to \vec{K}$

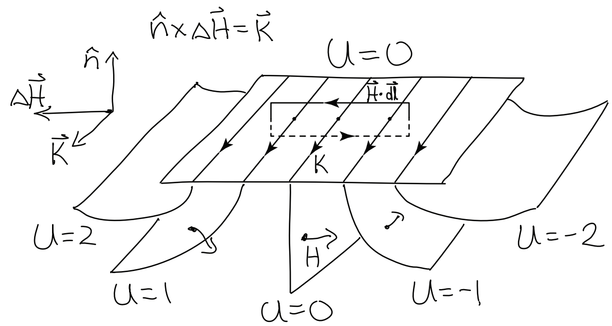
$$\mathcal{E}_H = \oint \vec{H} \cdot d\vec{l} = (H_{2t} - H_{1t}) l = K_s l = I \qquad \boxed{-\Delta U = \mathcal{E}_H = I}$$
$$= \int -\nabla U \cdot d\vec{l} = \int -dU = -\Delta U$$

~ electric

	vector	component	potential
$\mathcal{E}_E = 0$	$\hat{n} \times \Delta \vec{E} = 0$	$E_{2t} = E_{1t}$	$\Delta V = 0$
$\Phi_D = Q$	$\hat{n} \cdot \Delta \vec{D} = \sigma$	$D_{2n} - D_{1n} = \sigma$	$-\Delta \varepsilon \frac{\partial V}{\partial n} = \sigma$

~ magnetic

	vector	component	potential
$\mathcal{E}_H = I$	$\hat{n} \times \Delta \vec{H} = \vec{K}$	$H_{2t} - H_{1t} = K_s$	$-\Delta U = I$
$\Phi_B = 0$	$\hat{n} \cdot \Delta \vec{B} = 0$	$B_{2n} = B_{1n}$	$\Delta \mu \frac{\partial U}{\partial n} = 0$



~ surface current flows along U equipotentials ~ U is a SOURCE potential

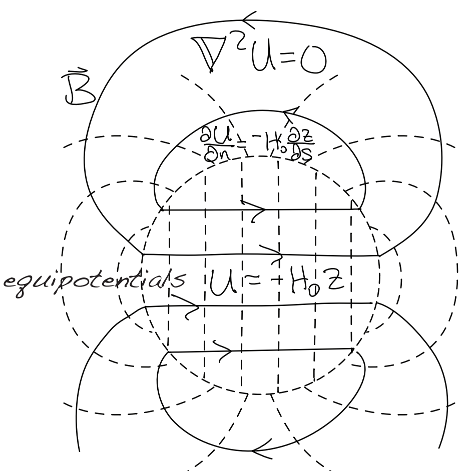
~ the current $I = I_2 - I_1$ flows between any two equipotential lines $U = I_1$ and $U = I_2$

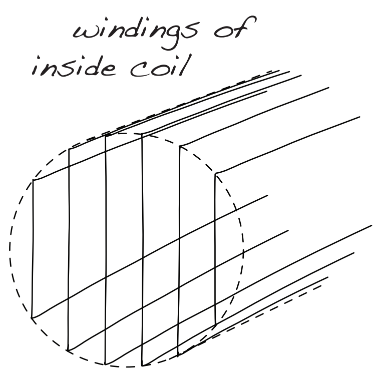
Scalar Potential Method

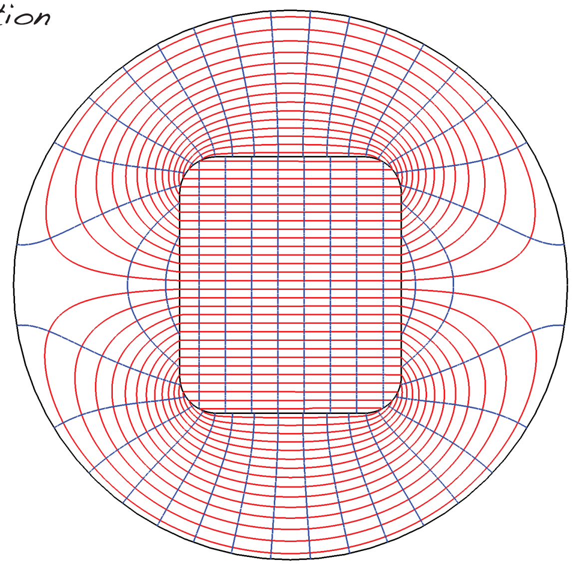
* procedure for designing a coil based on required fields and geometry:
  - ~ solve $\nabla^2 U = 0$ with flux boundary conditions from known external fields
  - ~ draw the equipotential CURVES on the boundary to form the windings (wires)
  - ~ current through each wire = difference between adjacent equipotentials

* utility of electric and magnetic potentials – direct relation to physical devices
  - ~ it is only possible to control electric potential, NOT charge distribution in a conductor
  - ~ conversely, it IS easy to control current distributions (by placement of wires) but this is related to the magnetic scalar potential

* example: cos-theta coil
  - ~ analog of cylindrical (2-d) electric dipole
  - ~ longitudinal windings, perfectly uniform field inside
  - ~ solve Laplace equation with flux boundary condition
  - ~ double cos-theta coil B=0 outside outer coil







* comparison of electrical and magnetic components

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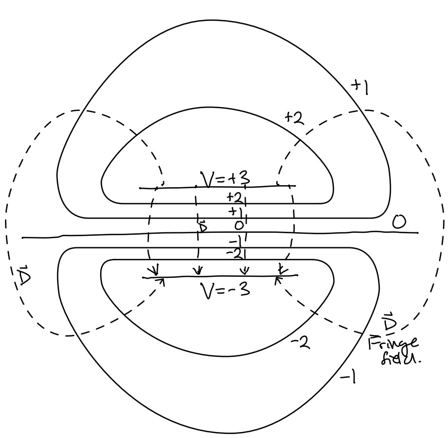
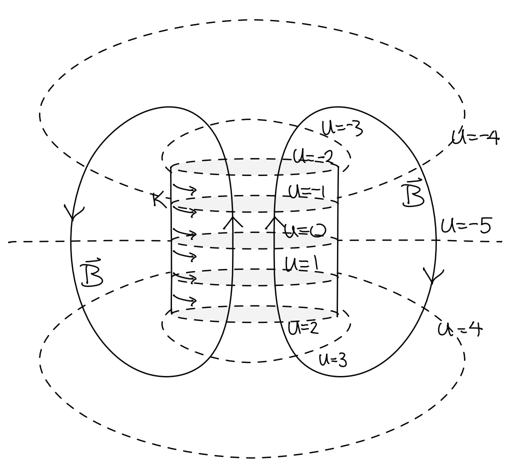


SOLENOID



TOROID