Section 6.4 – Magnetic Media

$\vec{m} = \beta \vec{B}$
$\vec{M} = \chi_m \vec{H}$

$\vec{p} = \alpha \vec{E}$
$\vec{P} = \chi_e \varepsilon_0 \vec{E}$

* constitutive relations: *magnetic susceptibility and permeability*

$$\varepsilon_0 \vec{E} = \vec{D} - \vec{P} \qquad \vec{D} = \varepsilon_0 (\vec{E}) + \vec{P} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

$$\frac{1}{\mu_0} \vec{B} = \vec{H} + \vec{M} \qquad \vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \underbrace{\mu_r}_{K_m} \vec{H} = \mu \vec{H}$$

$0 \le \mu_r \le$ "∞"

* linear and nonlinear media: $\vec{P}(\vec{E}, \vec{r}, \omega, T, \ldots)$ $\quad \vec{M}(\vec{B}, \vec{r}, \omega, T, \ldots$ history!$)$

$-1 \ll \chi_m \le \infty$

~ linear $\quad \vec{M} = \overleftrightarrow{\mu} \cdot \vec{H} \quad \overleftrightarrow{\mu}(\vec{H}) = \text{const.}$ $\quad$ *permeability independent of field strength*

~ isotropic $\quad \overleftrightarrow{\mu} = \mu \cdot \overleftrightarrow{I} \quad \vec{M} \parallel \vec{H}$ $\quad$ *same permeability in all directions*

~ homogeneous $\quad \mu(\vec{r}) = \text{const.}$ $\quad$ *same permeability throughout material*
still dicontinuities at boundary

* Gaussian units (CGS) $\quad \mu_0 = \varepsilon_0 = 1 \quad$ so $\quad \mu = \mu_r \quad \varepsilon = \varepsilon_r$

~ [H] = Oersted, [B] = Gauss ~ 0.0001 Tesla

~ Units of E and B also the same! $\quad \vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$

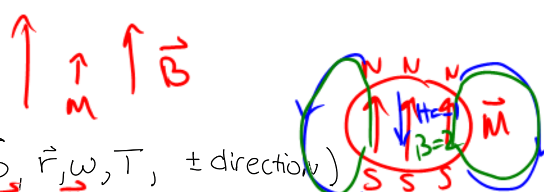
* diamagnetism

~ most similar to electiric $\quad \chi_m < 0 \quad \sim -10^{-5}$

~ useful for levitation

~ superconductor (SC) $\quad \chi_m = -1$! $\quad \vec{B} = 0$

* paramagnetism $\quad \chi_m > 0 \quad \sim 10^{-5} - 10^{-1}$



* ferromagnetism $\quad \mu_r \gg 1 \quad \Delta\vec{B} = \mu_\Delta \Delta \vec{H}(\vec{H}, \vec{B}, \vec{r}, \omega, T, \pm \text{direction})$

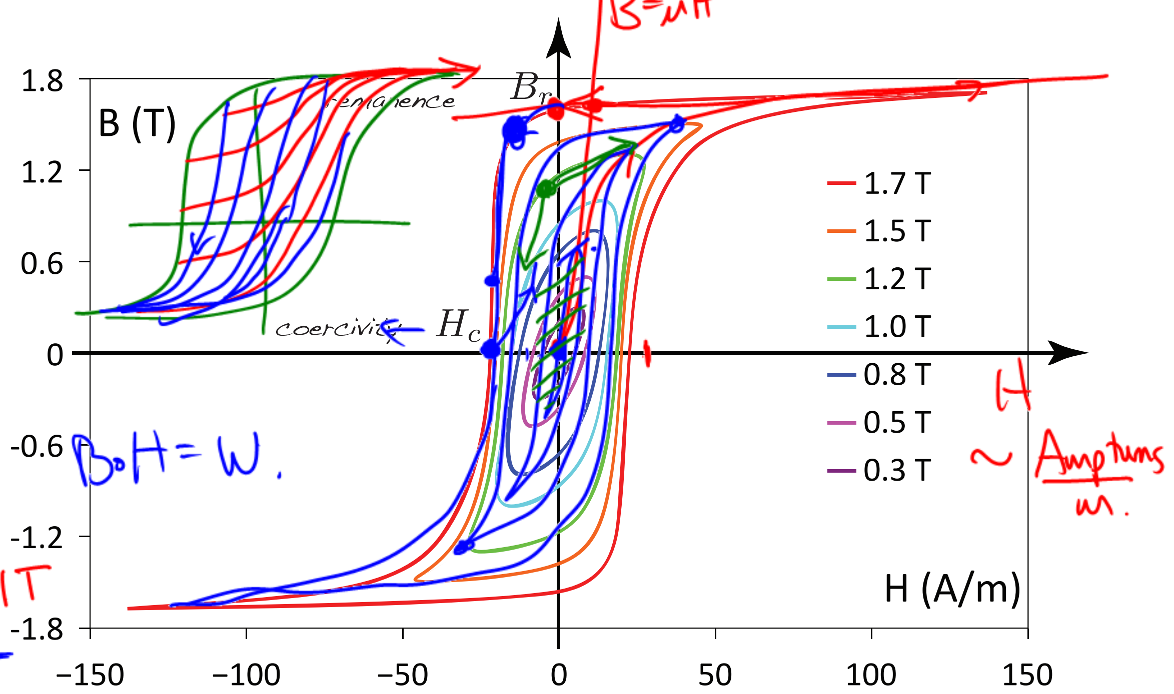
$\vec{B} = \mu \vec{H}$

~ electromagnet

~ iron-core transformers

~ μ-metal

~ 10^3 reduction in field.



$B = \mu H$

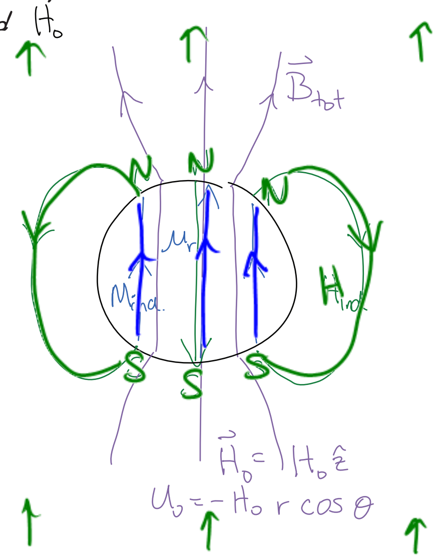
$\frac{B}{\mu_0} = H + \underbrace{\chi_m H}_{\max \approx 1\,T}$

http://en.wikipedia.org/wiki/Hysteresis

compare #4.7

$E \to H \quad \varepsilon_0 \to \mu_0$
$P \to \mu_0 M \quad \varepsilon_r \to \mu_r$
$D \to B$

* *Example 3: Magnetic permeability (linear homogeneous isotropic material)*
Permeable sphere in an external constant field H_0



$\vec{M} = \chi_m \vec{H}$ encapsulated in $\vec{B} = \mu\vec{H}$ see Ex 4.7 p186

$\nabla\times\vec{H} = \vec{J}_{tot} = 0$ everywhere $\rightarrow \vec{H} = -\nabla U$
$\nabla\cdot\vec{B} = -\nabla\cdot\mu\nabla U = 0 \rightarrow \nabla^2 U = 0$
BC's: $U(\infty)\to -H_0 z$, $\Delta U = 0$, $\Delta\mu\frac{\partial U}{\partial n} = 0$

$$U_1 = \sum_{l=0}^{\infty}\left[A_l (r/a)^l + \cancel{B_l} (a/r)^{l+1}\right] P_l(c_\theta)$$

$$U_2 = \sum_{l=0}^{\infty}\left[C_l (r/a)^l + D_l (a/r)^{l+1}\right] P_l(c_\theta)$$

BC#0: $\lim_{r\to\infty} U_2(r) = \sum_{l=0}^{\infty} C_l (r/a)^l P_l(c_\theta) = -H_0 r c_\theta \qquad C_1 = -H_0 a$
we exclude all multipoles except $l=1$, since the source $\vec{H}_0$ is pure dipole.

BC#1: $U_2|_a - U_1|_a = (-H_0 a + D_1 - A_1) = 0 \quad \Rightarrow D_1 = A_1 + H_0 a$

BC#2: $-\frac{\partial U_2}{\partial r}\Big|_a + \mu_r \frac{\partial U_1}{\partial r}\Big|_a = \left(H_0 + D_1\cdot 2/a + \mu_r A_1/a\right)\cos\theta = 0$

$3H_0 + (2+\mu_r)A_1/a = 0 \qquad A_1/a = \frac{-3}{2+\mu_r}H_0 \qquad D_1/a = \frac{-1+\mu_r}{2+\mu_r}H_0 = \frac{\chi_m}{2+\mu_r}H_0$

$U_1 = U_0 + \frac{\chi_m}{2+\mu_r}H_0 z = \frac{-3}{2+\mu_r}H_0 z \qquad \vec{H}_1 = \frac{3}{2+\mu_r}\vec{H}_0$
$U_2 = U_0 + \frac{\chi_m}{2+\mu_r}H_0 a^3\frac{\cos\theta}{r^2}$ where $U_0 = -H_0 z$

* compare with Ex#1: $U_1 = U_0 + \frac{1}{3}Mz \qquad U_2 = U_0 + \frac{1}{3}a^3 M\frac{\cos\theta}{r^2}$

$\vec{M} = \frac{3\chi_m}{2+\mu_r}\vec{H}_0 = \chi_m\left(\frac{3}{2+\mu_r}\right)\vec{H}_0 = \chi_m\vec{H}_1$ as dictated by $\vec{M} = \chi_m\vec{H}$

$\vec{B}_1 = \frac{3\mu}{2+\mu_r}\vec{H}_0 \qquad \vec{B}_2 = \mu_0\vec{H}_2 = \mu_0\left(\vec{H}_0 + \frac{Q\vec{m}}{4\pi r^3}\right)$ where $\vec{m} = \frac{4}{3}\pi a^3\cdot\frac{3\chi_m}{2+\mu_r}H_0$
$Q\vec{m} = 3\hat{r}\hat{r}\cdot\vec{m} - \vec{m}$