Section 7.2.3 - Inductance

* review: 3 "Ampere" laws
  ~ will use all 3 today

$$\nabla\times\vec{H}=\vec{J} \qquad \nabla\times\vec{A}=\vec{B} \qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\mathcal{E}_H=\Phi_J=I \qquad \mathcal{E}_A=\Phi_B \qquad \mathcal{E}_E=-\frac{d\Phi_B}{dt}$$

* new Ohm's law
  $V = I Z$

$$\frac{\text{current} \rightarrow \text{flux}}{\text{time}} \rightarrow \text{voltage}$$

$$I = I_0 e^{i\omega t}$$

$$V=\frac{1}{C}\int I\,dt = I\frac{1}{i\omega C}$$

$$V = IR \quad = IR$$

$$V = L\frac{dI}{dt} \quad = I\,i\omega L$$

* Mutual/Self Inductance - application of Faraday's law

$$B_1=\frac{\mu_0 I_1}{4\pi}\oint_1\frac{d\vec{l}_1\times\hat{\mathscr{r}}}{\mathscr{r}^2} \qquad \Phi_2=\int_2\vec{B}_1\cdot d\vec{a}_2$$

$$\Phi_2=\int_2\left(\frac{\mu_0}{4\pi}\oint_1\frac{d\vec{l}_1\times\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\cdot d\vec{a}_2\, I_1 \equiv M_{21}I_1$$

$$\Phi_2 = M_{21}I_1 \qquad \Phi = LI$$

$$\mathcal{E}_2=-M_{21}\frac{dI_1}{dt} \qquad \mathcal{E}=-L\frac{dI}{dt}$$

$$M_{21}=\Phi_2/I_1=\mathcal{E}_A/I_1=\oint_2\underbrace{\left(\frac{\mu_0}{4\pi}\oint_1\frac{d\vec{l}_1}{\mathscr{r}}\right)}_{\vec{A}/I_1}\cdot d\vec{l}_2=\frac{\mu_0}{4\pi}\oint_1\oint_2\frac{d\vec{l}_1\cdot d\vec{l}_2}{\mathscr{r}}$$

~ property of material and geometry
~ "back" emf: voltage drop across L, opposes changes in the current

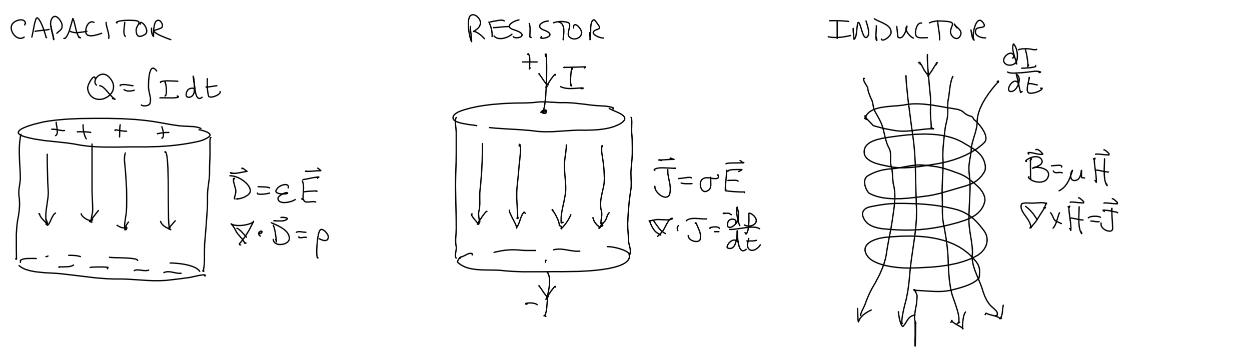
compare: $\vec{F}_{21}=-\frac{\mu_0}{4\pi}\oint\oint\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\vec{l}_1\cdot d\vec{l}_2\; I_1 I_2$

* Inductance matrix L
  ~ symmetric: mutual inductance
  ~ diagonal: self inductance

$$V_i=\sum_j L_{ij}\dot{I}_j \qquad M_{21}=M_{12} \qquad L_i\equiv M_{ii}$$

$$L=\begin{pmatrix} L_1 & M_{21} & M_{31} \\ M_{12} & L_2 & \ddots \\ M_{13} & \ddots & \ddots \end{pmatrix}$$

* three electrical devices - one calculation!



$$Q=\sigma_f A=\int\vec{D}\cdot d\vec{a}=\Phi_D \qquad I=\int\vec{J}\cdot d\vec{a}=\Phi_J \qquad NI=N\Phi_J=\mathcal{E}_H$$

$$V=\int\vec{E}\cdot d\vec{a}=\mathcal{E}_E \qquad V=\text{same}=\mathcal{E}_E \qquad V=-N\mathcal{E}_E=N\frac{d\Phi_B}{dt}$$

$$C=Q/V=\varepsilon\,\Phi/\mathcal{E}=\frac{\varepsilon A}{l} \qquad R=V/I=\mathcal{E}/\sigma\Phi=\frac{l}{\sigma A} \qquad L=V/\dot{I}=\Phi/I=N^2\mu\,\Phi/\mathcal{E}=\frac{N^2\mu A}{l}$$

* units

$[C]=F \qquad [R]=\Omega \qquad [L]=H$

$[\varepsilon_0]=F/m \qquad [\mu_0]=H/m$

Section 7.2.4 - Energy in the Magnetic Field

* example: L-R circuit

$$\mathcal{E} = IR + L\dot{I}$$

$$(\mathcal{E} - IR)\,dt = L\,dI = -(I - \mathcal{E}_0/R)\,dt \qquad \text{let } u = \mathcal{E} - IR, \quad du = -R\,dI$$

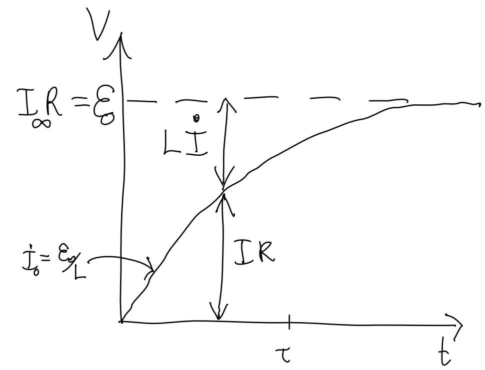
$$u\,dt = -L/R\,du$$

$$\ln(u/u_0) = \int \frac{du}{u} = -R/L \int dt \qquad \text{let } \tau = L/R$$

$$(\mathcal{E} - IR) = (\mathcal{E} - I_0 R)\,e^{-t/\tau} \qquad u = u_0 e^{-t/\tau}$$

$$I = \frac{\mathcal{E}}{R} - \left(\frac{\mathcal{E}}{R} - I_0\right)e^{-t/\tau} \qquad \text{where } \mathcal{E} = I_\infty R$$

$$= I_\infty - \Delta I\, e^{-t/\tau} \qquad \Delta I = I_\infty - I_0$$



~ time constant $\tau = L/R$

note: initial slope depends on L, not R
larger R just means lower I_∞

* work against back emf: "electrical inertia"

$$\frac{dW}{dt} = -\mathcal{E}I = LI\frac{dI}{dt} \qquad W = \tfrac{1}{2}LI^2$$

$$W = \tfrac{1}{2}I\oint \vec{A}\cdot d\vec{l} \qquad LI = \Phi_B = \mathcal{E}_A$$

$$= \tfrac{1}{2}\int_V \vec{A}\cdot\vec{J}\,d\tau \qquad \tfrac{1}{2}\vec{p}\cdot\vec{v} = \tfrac{1}{2}mv^2 \quad \text{energy from "potential momentum"}$$

$$= \frac{1}{2\mu_0}\int_V \vec{A}\cdot\nabla\times\vec{B}\,d\tau \qquad \nabla\cdot(\vec{A}\times\vec{B}) = (\nabla\times\vec{A})\cdot\vec{B} - \vec{A}\cdot\nabla\times\vec{B}$$

$$= \frac{1}{2\mu_0}\int_V B^2\,d\tau \qquad \int_{\partial V}\vec{A}\times\vec{B}\cdot d\vec{a} = \int_V B^2\,d\tau - \int_V \vec{A}\cdot\nabla\times\vec{B}\,d\tau$$

$$\Delta W = \tfrac{1}{2}\int \Delta\vec{B}\cdot\vec{H}\,d\tau$$

compare:

$$\Delta W = \tfrac{1}{2}\int \Delta V\,\rho\,d\tau$$

$$= \tfrac{1}{2}\int \Delta V\,\nabla\cdot\vec{D}\,d\tau$$

$$= \tfrac{1}{2}\int -\Delta\nabla V\cdot\vec{D}\,d\tau$$

$$= \tfrac{1}{2}\int \Delta\vec{E}\cdot\vec{D}\,d\tau$$

$$u = \tfrac{1}{2}(E\cdot D + B\cdot H)$$

* example 7.13

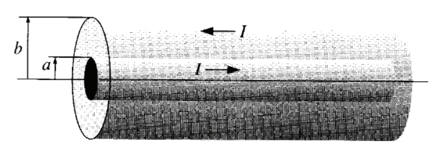


Figure 7.39

$$\vec{H} = \frac{I\hat{\phi}}{2\pi s} \qquad u = \tfrac{1}{2}\vec{B}\cdot\vec{H} = \frac{\mu I^2}{8\pi^2 s^2}$$

$$\tfrac{1}{2}\frac{LI^2}{l} = W/l = \int u\,da = \int_{s=a}^{b} 2\pi s\,ds\,\frac{\mu I^2}{8\pi^2 s^2} = \int_a^b \frac{\mu I^2}{4\pi}\frac{ds}{s} = \frac{\mu I^2}{4\pi}\ln\frac{b}{a}$$

$$L/l = \frac{\mu}{4\pi}\ln\left(\frac{b}{a}\right)$$