Arizona State University, Physics 311
Problem Set #7, Rev. A, due Wednesday, 2014-03-19

1. An infinite **cylindrical rod** of radius R centered in the xy-plane and extending along the z-axis has a uniform polarization $\boldsymbol{P} = P\hat{\boldsymbol{x}}$.

a) Calculate the bound surface charge density σ_b.

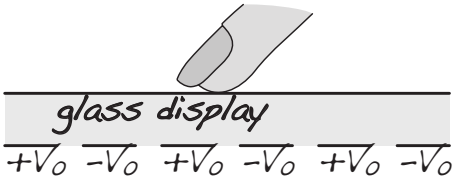
b) Calculate $\boldsymbol{E}_b$ (due to the bound charge) inside and outside the cylinder as a function of $\boldsymbol{P}$.

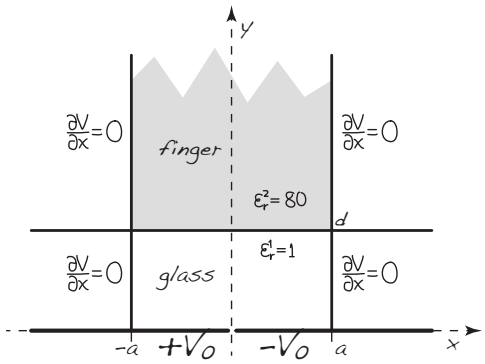
c) Suppose the rod is placed in an external electric field $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$, and that $\boldsymbol{P} = \chi_e\epsilon_0\boldsymbol{E}$, where $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_b$ is the total electric field inside the rod. Calculate the values of $\boldsymbol{E}$, $\boldsymbol{P}$, and $\boldsymbol{D}$ inside and outside the rod.

d) Show that $\epsilon_0 E_{2n} - \epsilon_0 E_{1n} = P_n$ and that $D_{2n} - D_{1n} = 0$ for your solution.

e) Solve the same problem (part c) as a boundary value problem with dielectric constant $\epsilon_r = 1 + \chi_e$ inside the rod and $\sigma_f = 0$ on the surface (only bound surface charge). Show that the two answers are consistent.

2. In a **capacitive touch-screen**, your dielectric finger modifies the capacitance between two strips of the transparent conductor indium tin oxide (ITO). We will model a single cell (from the middle of one strip to the middle of the next) of this display as an semi-infinite rectangle extending upward from the strips. Use the boundary conditions $\partial V/\partial x = 0$ on the sides at $x = \pm a$ to ensure the correct periodic symmetry.





a) Solve a boundary value problem for the potential when a finger is touching the glass. Assume the glass thickness is d, and your finger of pure water ($\epsilon_r^2 \approx 80$) fills the whole region beyond the glass. Calculate the first two nonzero terms of the Fourier expansion. Set $\epsilon_r = 1$ to determine the solution with a finger touching the glass.

b) Plot the equipotentials and field lines in the region with and without your finger.

c) Calculate the capacitance/length with and without your finger, as a function of d/a. Don't forget fields on both the top and bottom of the strip. What glass thickness do you recommend?

d) If the voltage on each wire is $\pm V_0 = \pm 2V$, what difference in charge must the device be able to measure? ($d = a = 3$ mm, 1 cm long) If the strips were joined into an LRC circuit, what frequency shift $\Delta\omega$ would the device need to discriminate?

Also, Griffiths 3ed[4ed] Ch. 4, #8[8], 16[16], 24[24], 29[29], 33[36].