Arizona State University, Physics 311
Problem Set #11, Rev. A, due Wednesday, 2014-04-30

1. In a **rail gun**, a conductive projectile of mass m bridges the gap between two conducting rods of radius a and length l, separated by a distance d. Each rail is connected to one terminal of a low-inductance capacitor C, with charge Q, initially at voltage V_0. At $t = 0$, the projectile is placed touching both rails at the end near the capacitor, completing the circuit. The projectile is accelerated by the magnetic force on the current I passing through it. The goal of this problem is to calculate the velocity v of projectile as it reaches the far end of the rails. Assume that the total resistance R is dominated by the contact resistance between the projectile and the rails, and that the inductance is dominated by the rails (a single turn coil).

a) Calculate the magnetic force on the projectile as a function of current I in the rails. Calculate the power expended in accelerating the projectile. Model the lost power with an effective voltage drop $V_b = P/I$ (back EMF) in the circuit.

b) Calculate the inductance as a function of x, the position of the projectile along the rail. Model the rail as an LRC circuit to calculate dI/dt as a function of Q, I, x, and v.

c) Numerically integrate dI/dt of part (b) and $F = ma$ to determine the position $x(t)$ of the projectile and charge $Q(t)$ in the capacitor as a function of time. Choose reasonable physical values for V_0, C, R, m, d, and a.

A simple, yet effective, way to do this is in a spreadsheet with columns for t, Q, I, x, and v. Put the initial values in the first row. Calculate each subsequent row at step Δt forward in time using values from the previous row. For example, $Q_{i+i} = I_i \Delta t$ and $v_{i+1} = F(I_i)\Delta t/m$. Copy the cells (formulas) from one row to the next.

d) Calculate the ratios of kinetic energy of the projectile versus energy dissipated in the resistor, energy remaining in the capacitor, and inductive energy, when the ciruit is broken as the it leaves the rails. What happens to the inductive energy?

e) Bonus: Tune the capacitance as a function of the other parameters to maximize the fraction of kinetic energy transferred to the projectile.

2. A **quarter shrinker** can contract a quarter of initial diameter 24 mm and thickness 1 mm to almost half its radius. Of course mass is conserved, so the quarter will be twice as thick. The quarter is placed in an 11 turn (33 mm long) solenoid of 3 mm diameter wire, with a winding diameter of 30 mm. The solenoid is connected to a low-inductance high-voltage capacitor C. Assume that the inductance L and resistance R is dominated by the coil, and that the resistivity of Cu is $\rho = 1.678\ \mu\Omega\,\mathrm{cm}$.

a) Calculate R and L of the coil. Neglecting the mutual inductance of the quarter, calculate the current $I(t)$ through the coil as a function of time.

b) Calculate the induced current density in the the quarter as a function of time. Using the Lorentz force law, calculate the contractive force on the quarter. What do you think will happen? See http://capturedlightning.com for the answer!

Also, Griffiths 3ed[4ed] Ch. 7, #14[14], 20[22], 30[32], 31[34], 42[44], 48[50], 53[57], 54[58].