Syllabus for PHY 311 Spring 2014

Classical Particles, Fields, and Matter II

Class schedule:	M W F 10:30–11:20, PSF123
Instructor:	Christopher B. Crawford, PSF430, crawford@pa.uky.edu, 480-965-2238
Office hours:	open office policy; see homepage for schedule
Homepage:	http://www.pa.uky.edu/∼crawford/phy311_sp14
Textbook:	David J. Griffiths, "Introduction to Electrodynamics," 3rd or 4th Ed. (required)
	Murray R. Spiegel, "Theory and Problems of Vector Analysis," (recommended)

Course Description The goal of this course is to become proficient in the techniques of classical field theory within the context of its archetype, the electromagnetic field. This new approach to electromagnetism a) provides a concrete example with which to identify the basic principles of field theory, b) gives new insight into the significance of and relations between electromagnetic quantities, and c) provides powerful methods for solving practical problems involving voltage and current in continuous dielectric, magnetic, and conductive media. The course focuses on the development of a qualitative physical intuition of abstract electrodynamic concepts, and applying these principles to solve real-life problems both analytically and numerically.

A common theme of the course is the longitudinal/transverse (L/T) separation of fields, encoded by the following projection identities (which illustrate the geometric meaning of the dot and cross product):

$$\begin{aligned} \boldsymbol{v} &= \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}\cdot\boldsymbol{v} - \hat{\boldsymbol{n}}\times\hat{\boldsymbol{n}}\times\boldsymbol{v} &&= (P_\parallel + P_\perp)\boldsymbol{v}. \\ \nabla^2 &= \nabla\nabla\cdot \quad - \nabla\times\nabla\times &&= \nabla_\parallel^2 + \nabla_\perp^2 \end{aligned}$$

This is the basis of the Helmholtz theorem and Poincaré lemma, which are at the heart of Maxwell's equations and their potential formulation, respectively. The theme of L/T field components continues in the form of the integrals for *flux* $\varPhi_F \equiv \int_S \boldsymbol{F}\cdot d\boldsymbol{a}$ and what we call *flow*, $\mathcal{E}_F \equiv \int_C \boldsymbol{F}\cdot d\boldsymbol{l}$. These integrals embody the two complementary characteristics of a vector field, identified by its unique sources by means of Gauss' and Stokes' theorems, respectively. They also relate to the two complementary geometrical representations of the field: field (flux) lines and equipotential (flow) surfaces, which represent a unit of flux through any surface or a unit of flow along any path, respectively. The geometrical significance these deep field theoretic theorems will be presented graphically, borrowing from the imagery of differential forms.

This course will present a unified geometric view of electrostatics and magnetostatics in terms of this L/T separation. One component derives from the source of the field, while the other representation controls its dynamical aspects (force, potential energy and momentum). The *source* and *force* nature of these fields will be developed in five independent formulations of both electrostatics and magnetostatics. The connections between each formulation stem from the fundamental theorems mentioned above. In fact, a key difference between electric and magnetic fields is the reversal between the roles of source and force played by the L/T representations. In this sense, the electric and magnetic fields will be constructed as the L/T projections of the single Faraday tensor $F_{\mu\nu}$ in space-time, derived from a combined vector and scalar potential, the 4-vector A_μ, and with charge and current sources unified into a single conserved current J^μ. The addition of the gauge transformation symmetry λ of the potential A_μ completes the circle, being related to the conserved source current J^μ via Noether's theorem. In analogy to the standard 'force potentials', we develop 'source potentials' and show their immediate practicality in the case of the magnetic scalar potential U. The fundamental structure of the electromagnetic field described herein is captured by these two coupled exact sequences (derivative chains) of gauge, potentials, fields, and currents:

$$\begin{tikzcd} \lambda \arrow[r, "d"] & (V,\boldsymbol{A}) \arrow[r, "d"] & (\boldsymbol{E},\boldsymbol{B}) \arrow[r, "d"] \arrow[d, shift left, "\epsilon"'] \arrow[dr, "\sigma"] & 0 & \text{(force)} \\ (\boldsymbol{C},U) \arrow[r, "d"] & (\boldsymbol{D},\boldsymbol{H}) \arrow[r, "d"] \arrow[u, shift left, "\mu"'] & (\rho,\boldsymbol{J}) \arrow[r, "d"] & 0. & \text{(source)} \end{tikzcd}$$

The electrostatic Poisson equation generalizes to the wave equation $-\Box^2 A^\mu = -\mu J^\mu$, treated in the continuation course PHY 412: "Classical Particles, Fields, and Matter III".

The fifth formulation of electrostatics, Poisson's equation $-\nabla \cdot \epsilon \nabla V = \rho$ for the electric potential V, is emphasized as a practical tool for solving realistic electrostatic problems. These boundary value problems are solved analytically using separation of variables and Sturm-Liouville theory, in the context of infinite-dimensional linear function spaces, or numerically using the Finite Element method. We will also explore a general formulation of the multipole expansion for both near- and far-field approximations. For magnetostatics, we derive the analogous formulation of Poisson's equation $-\nabla \cdot \frac{1}{\mu} \nabla \boldsymbol{A} = \boldsymbol{J}$ for the magnetic vector potential $\boldsymbol{A}$. However for the practical calculations, we develop a powerful unpublished technique for actually calculating the windings of a magnetic coil required to produce the desired magnetic field. This technique uses the Laplace equation $-\nabla \cdot \mu \nabla U = 0$, based on a practical intuitive interpretation of the magnetic scalar potential U, which naturally 'flows' (no pun intended) out of our geometric understanding of the structure of the electromagnetic fields.

Attendance Although you have studied both electromagnetism and vector calculus in the past, don't be fooled! This course will employ a level of mathematical sophistication that will be challenging for even the most prepared student. It will take hard work, dedication, and teamwork to master this material. Lectures and homework extend beyond material covered in the textbook, so the reading the assigned sections ahead of class is essential for understanding the lectures. Please prepare for class with questions and comments from your reading and share what you have learned with others during class; this takes priority over the prepared lecture. There is no credit for attendance; however, students are responsible for all material covered in class and in the textbook.

Office Hours I am committed to helping you succeed if you are willing to invest the necessary effort. I have an open door policy; come by my office and discuss physics anytime unless my door is closed (for a phone conference or approaching deadline). Please turn off cell phones and text messaging in my office and class, and prepare for your visit by reading the relevant textbook material. I will hold an optional one hour homework recitation each week.

Grading The course material is strongly cumulative, and so it is impossible to dismiss misunderstandings and move on. If you are falling behind, please seek immediate assistance from your instructor or classmates. There are three exams covering Chapters 1-2, 3-4, and 5-6. The final exam is cumulative. Exams are only rescheduled for officially excused absences. Extra credit is awarded for finding new errors in the textbook, or solving special questions posed during class. Homework includes standard textbook problems (credit for completion) and supplemental problems (graded). Students are encouraged to study and discuss homework together, but must turn in their own work (see below). Homework is due in class, in my office, or digitally on Blackboard by 6:00 AM the morning after the assigned due date. Unexcused late homework will receive 50% for completeness.

Grade breakdown	
homework	30%
exams	3×15%
final exam	25%
TOTAL	100%

Letter grade	
±A	80–100%
±B	65–79%
±C	50–64%
±D	40–49%

Academic Conduct Copying homework or exams from people, solution manuals, online, or any other source is plagiarism and will not be tolerated. It is not fair to yourself or other students. University policies and procedures regarding cheating and other academic conduct will be strictly adhered to and can be reviewed at http://provost.asu.edu/academicintegrity/policy/StudentObligations.

Academic accommodations due to disability If you have a documented disability that requires academic accommodations, please see me in the first two weeks to address it.