"Without data you are just another opinion" – Anon.

A. Crater formation in the laboratory

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We demonstrate how measurements of the diameter of a crater formed by dropping a small steel ball into a sand-filled container allow one to deduce the functional dependence of the crater diameter on the kinetic energy of the falling ball.

A1. INTRODUCTION

Craters are found on all planets except Jupiter, which has no solid surface, and moons of the solar system, and an understanding of crater formation is important for examining the history of the solar system. On Earth, craters are formed by meteor impact as well as by large-scale underground explosions, e.g., nuclear weapons.[1] In either case, the energy:mass ratio is very large (compared to chemical explosions, for example), and it is believed that crater formation by either process follows the same scaling laws. According to Gault et al.,[2] the kinetic energy of an impacting meteor is distributed among five processes: heating, comminution (the creation of new surface area), deformation, ejection of material, and seismic waves.

If plastic deformation is the most important process, then the volume V of the crater must scale with the energy of the meteor. Since $V \sim D^3$, where D is the crater diameter (as shown in the figure), then D scales as the cube root of the energy, or

$$D \sim E^{1/3} \tag{1}$$

On the other hand, if material ejection absorbs most of the energy, then the kinetic energy is converted to gravitational potential energy needed to lift a volume $V \sim D^3$ to a height approximately equal to the crater depth. Since the depth is proportional to D (for a spherical crater), then

$$D \sim E^{1/4} \tag{2}$$

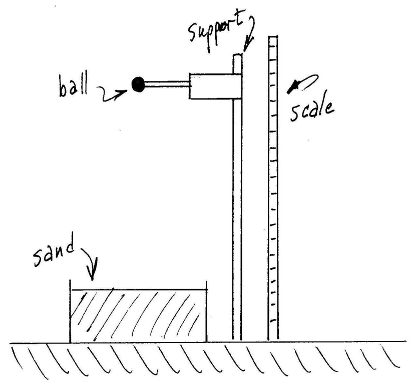
A2. APPARATUS AND PROCEDURE

The experimental apparatus is arranged as shown in the figure. A container is filled to a depth of about 8 cm with dry sand. The sand-filled container is placed on the floor close to a lab workbench. A standard table clamp and vertical pole support a 2-m stick and a ball launcher. The latter (a three-pronged spring-loaded lens holder works nicely) is centered directly above the sand. To simulate cratering, a small steel ball of known mass m and diameter d is dropped from a known height h into the sand, creating a well-defined circular crater of diameter D. The crater diameter is measured to the nearest mm using calipers. Between trials, the sand is "sifted" by plunging a piece of stiff hardware mesh into the sand repeatedly. Following this, the sand surface is leveled by gently shaking the container horizontally.

The immediate task is to determine the crater diameter as a function of the kinetic energy E of the falling ball. Using balls covering a range of diameters, and appropriately chosen heights, the value of E can be varied over approximately two orders of magnitude. For each ball and height chosen, several repeated measurements are collected so as to establish meaningful estimates of both the mean diameter and its uncertainty.

1. J. Mydosh, *Impact Cratering* (Oxford U.P., New York, 1989).

2 Donald E. Gault et al., "Some comparisons of impact craters on Mercury and the Moon," J. Geophys. Res. 80, 2444–2460 (1975).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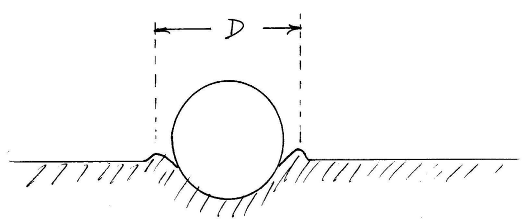
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B. Crater analysis

B1. Analysis Review

Recall some of the main ideas of data analysis we have discussed previously: Given an infinite set of measurements x_i subject to random error, these date are distributed symmetrically about the 'true' value x_{true}, and 68% of the measurements fall within 1σ, the standard deviation, of x_{true}. A finite set of n measurements $x_1, x_2, ..., x_n$ (the data sample) can be interpreted as a subset of the infinite data set; the purpose of repeated measurements is to obtain sufficiently accurate estimates of x_{true} and σ. The best estimate of x_{true} is the weighted average of the data sample, and the best estimate of σ is the sample standard deviation s, defined in the usual way. Since the sample standard deviation is itself a random variable, our ability to estimate σ from the data (using the calculated value of s) improves with increasing number of measurements n. Finally, we recall the standard deviation of the mean,

$$\sigma_\mu = \frac{\sigma}{\sqrt{\mathrm{n}}}$$

as the best measure of the overall uncertainty of the full experimental data set. In this expression, σ represents the standard deviation of each trial measurement, while σ_μ is the error made in determining our best estimate of the mean of the parent distribution.

B2. Data Sets

Eight steel balls with differing masses were dropped into a sand tray a total of six times each. At each drop, the diameter of the resulting crater was measured. The table below gives the mass and the fall distance for each ball. The columns to the right in the table list the set of six measured crater diameters.

Data Set A

Mass (g)	Height (cm)	Crater Diameters (mm)					
8.5	52.6	52	52	27	35	34	50
28.4	52.8	76	60	70	68	54	67
225.7	53.0	99	120	106	121	102	111
8.5	100.2	54	37	46	60	54	45
28.4	100.5	69	76	65	83	68	90
225.7	101.3	115	127	111	118	119	128
225.7	199.3	156	151	144	155	155	149
28.4	199.8	93	92	79	95	82	83

Two additional data sets were collected to investigate the question of whether the ball diameter explicitly affected the diameter of the resulting craters. Twenty five ball drops were observed for both a relatively large and a relatively small steel ball, but the drop heights were adjusted so that both arrived at the sand with the same energy. These measured crater diameters are listed below as data sets B and C.

Data Set B
49 51 33 37 43 48 57 44 58 51 57 48 41 33 48 54 44 42 44 37 42 45 47 46 52

Data Set C
48 63 49 63 40 51 44 52 52 27 35 34 50 60 44 54 52 37 50 40 60 46 61 43 52

B3. Data Analysis

Your work on this project includes several components, each of which is listed below. You should submit a legible and clearly labeled paper copy of your solutions to each of these components. Your word answers to the questions should be given in complete sentences. Math calculations should proceed logically, with equal signs (=) used to show the steps of your work. Tables and plots should follow the general formats which were previously discussed in class. Use the codes you have prepared in your ToolBox to do the calculations and make the plots. Remember that you may not use the statistical libraries in Python for these calculations. (If you do, you will not get the correct answer. Really!) Of course, you will need to report the errors in any final numerical results.

a) For Data Set A: Construct a table listing the energy of the falling ball, and the mean, standard deviation, and standard deviation of the mean (standard error) of the measured crater diameter D. **Do not** list every ball drop that was observed, just the composite result at each energy. Indicate how many data points (drops) were collected at each value of energy. You should determine the mean value of D and its error for each of eight different values of E. Also include columns in your table with the values of log(D) and its error.

As you construct your table, make sure to pick appropriate units for each column of numbers, and label the columns with these units. For example, don't report measured lengths of a few cm in units of meters.

b) Do the measured values of D in Data Set B quantitatively display Normal, i.e. Gaussian, errors? Hint: Calculate χ^2 for a comparison between the distribution of measured diameters in Data Set B and a suitably normalized and parameterized Gaussian function. Make a point plot of the binned data, and overlay a curve of the Gaussian function. Show the error bars for each measured bin yield. Use a bin width of 2 mm for analyzing and plotting these data.

c) Data Sets B and C were collected at the same energy, but with different ball sizes. Do these data support the hypothesis that the crater diameter is not an explicit function of ball size? Your answer to this question requires a quantitative analysis, starting with a calculation of the linear correlation coefficient.

d) Do a least-squares fit of your results from Data Set A to determine the scaling exponent and its associated uncertainty. *You must not use a 'canned' program to do this fit. Write your own Python program, or use the one already in your ToolBox.* Plot the fit and data together on a log-log plot.

e) Which model of crater formation does your data support? What is the estimated probability that your conclusion is wrong?

B4. Report Format

Your final report on this project should be presented as a brief technical report, not as a homework assignment.

Imagine that you are employed by Boeing under contract to NASA, which wants to characterize potential lunar landing sites. Your Group Leader at Boeing doesn't like to read long reports, but she has given you this assignment to complete as part of Boeing's overall work on the NASA project. She like succinct and meaningful text answers, and informative, easy-to-read plots. She also likes complete sentences and logical presentations of math equations. She does not ask you to type your reports, but she does require them to be clean and easy to read.

This scenario is not too far-fetched, in that it demonstrates the kind of assignment which a physicist is often asked to do in an industrial research setting.

Finally, here are few additional Python commands which might be useful ...

plt.yscale('log') gives y-axis a log scale. also works for x-axis
plt.xlim(a,b) sets the plotting limits of the x-axis. also works for y-axis
plt.tight_layout() prevents overlap of figure labels when putting multiple plots on a page