

**University of Kentucky, Physics 335**  
**Laboratory #2, Rev. A, due Wednesday, 2023-09-06**

The purpose of this lab is to *theoretically* and *experimentally* quantify the distribution of the random variable  $X$  representing the number of heads from 10 coin tosses. Perform and analyze the experiment in pairs, but submit your own separate assignment with all plots and calculations.

**1. Theoretical estimation of the Parent Distribution**

a) How many possible outcomes are there from  $n = 10$  coin tosses? Tabulate the binomial distribution for  $n = 10$  and  $p = 1/2$  using Pascal's triangle and the formula  $\binom{n}{k} = n!/k!(n-k)!$  (omit the divisor  $N = 2^{10}$  in the table).

b) Draw a step plot of the normalized distribution  $P(x)$ , labeling your axes in the figure, which should span the entire width of your sheet.

c) Calculate the mean  $\mu = np$  and standard deviation  $\sigma = \sqrt{np(1-p)}$  from both the parent distribution the formulas. Plot the mean as a vertical line on the distribution and indicate the  $1\sigma$  interval in each direction with parenthesis on the abscissa.

**2. Experimental measurement of a Sample Distribution**

a) Perform an experiment to estimate  $P(x)$  by tossing  $n = 10$  coins  $N = 25$  times and recording the number of heads  $x_i$  after each throw labeled by  $i = 1, \dots, N$ . Calculate the mean  $\bar{x}$  and standard deviation  $s$  of this sample. Plot each point  $x_i \pm \delta x$  with error bars versus  $i$  on the abscissa, where the uncertainty  $\delta x$  is taken to be the standard deviation. Draw a horizontal line at height  $\bar{x}$  through all points. Plot the parent mean  $\mu$  and  $\pm 1\sigma$  ranges from 1c, now on the ordinate.

b) Add the number of occurrences of each value of  $x$  to a new line of the *frequency table* of 1a). Normalize the distribution to 1 and plot  $(h_x \pm \sqrt{h_x})/N$  with a solid circle and error bars in each bin on the graph of 1b). Draw a horizontal error bar above the abscissa representing the mean value  $\bar{x} \pm \delta\bar{x}$ , where  $\delta\bar{x} = s/\sqrt{N}$ . We will justify both square roots separately later on.

**3. Calculation of the Combined Distribution**

a) Tabulate your frequency distribution, mean, and standard deviation on the white board along with each other group. Calculate the combined sample distribution, its mean (compared with the mean of the means  $\bar{\bar{x}}$ ), and standard deviation  $\bar{s}$  (note the standard deviation is averaged *in quadrature*), and add it to a new line in your frequency table of 1a).

b) Add the combined distribution  $(h_x \pm \sqrt{h_x})/N$  from 3a) to the plot of 1b) using unfilled square makers with error bars. Plot the mean  $\bar{x}_i$  from each group with ticks on the horizontal error bar of 2b). Add a horizontal error bar for the combined mean  $\bar{\bar{x}} \pm \delta\bar{\bar{x}}$  above the previous one.

c) Qualitatively compare the plots/statistics of the parent, sample, combined distributions.

4. Suppose that some groups are “sloppy” when they tossed their coins in step 2a), for example, they toss all 10 coins at once, and most of the coins flip together rather than independently. What kind of signal would this “flipping together” behavior have on the data you collected? Or in other words, what would you look for to see if this happened to your group? (You do not need to perform this analysis, just describe what you are looking for.)