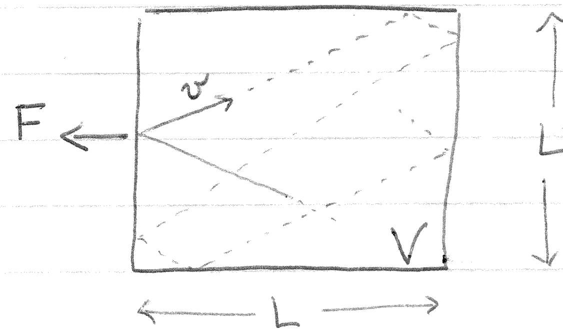
Brief overview of Kinetic Theory

Q:- How can you increase your maximum force, ie. to break open a door?

- what about rolling a train car from rest?



$$P \cdot V = N \cdot \underbrace{k}_{} \cdot T \quad (\text{or } nRT)$$

$$P \equiv \frac{F}{A} = \left\langle \frac{\Delta p}{\Delta t} \right\rangle \frac{N}{A} \quad \boxed{\text{Newton's Law}}$$

$$\Delta p = 2 m v_x$$
$$2L = v_x \Delta t$$
$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$
$$v^2 = v_x^2 + v_y^2 + v_z^2$$
$$KE = \tfrac{1}{2} m \langle v^2 \rangle \quad \text{single atom}$$

$$= m \langle v_x^2 \rangle \frac{N}{V}$$

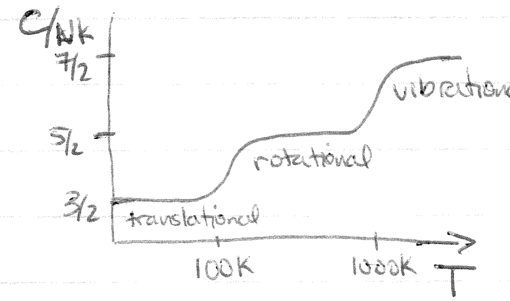
$$P \cdot V = N \cdot \underbrace{\tfrac{1}{3} m \langle v^2 \rangle}_{kT} \quad \boxed{\text{Pressure Law}}$$

- what is k? what is temperature?

$$KE = \underbrace{3}_{\#\text{ degrees of freedom}} \cdot \tfrac{1}{2} kT \quad \boxed{\text{equipartition theorem}}$$

$$C_v = \left(\frac{dE}{dT}\right)_V = \tfrac{3}{2} Nk \quad \left(\text{monatomic gas}\right)$$

constant volume heat capacity.



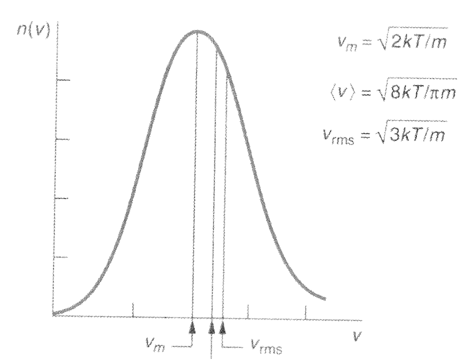
Maxwell - Boltzman Distribution

$$\underbrace{n(\varepsilon)}_{} = \underbrace{g(\varepsilon)}_{} \cdot \underbrace{f(\varepsilon)}_{}$$

$n(\varepsilon)$: # of particles of energy ε

$g(\varepsilon)$: # of states of energy ε
= density of states
= statistical weight.
= degeneracy.
= "phase space"

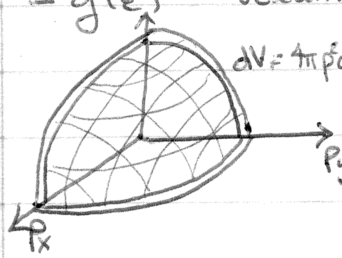
$f(\varepsilon)$: # of particles in each state of energy ε.
= distribution function.
= occupancy.

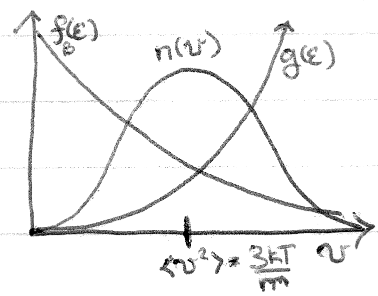


Density of States

$$g(\varepsilon) = dp_x \cdot dp_y \cdot dp_z = 4\pi p^2 dp \propto \boxed{v^2 dv}$$

- each $\vec{p}$ is considered a separate state.
- $g(\varepsilon)$ = "volume" in $\vec{p}$-space





Boltzman Distribution Function

$$\boxed{f_B(\varepsilon) = e^{-\varepsilon/kT}}$$

- excited states at low temperature are exponentially improbable.
- natural distribution of energy due to random collisions "statistical mechanic

Maxwell distribution:

$$n(v) = \underbrace{4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2}}_{\text{normalization}=C} \underbrace{v^2}_{\text{density of states}} \underbrace{e^{-\frac{1}{2}mv^2/kT}}_{\text{Boltzman distribution}}$$

$$\int_0^\infty n(v)\,dv = C\int_0^\infty v^2 e^{-\frac{1}{2}mv^2/kT}\,dv = C\,\frac{\sqrt{\pi}}{4}\left(\frac{m}{2kT}\right)^{-3/2} = \boxed{1}$$

$$\langle v^2 \rangle \equiv \int_0^\infty v^2 \cdot n(v)\,dv = C\int_0^\infty v^4 e^{-\frac{1}{2}mv^2/kT}\,dv = C\,\frac{3\sqrt{\pi}}{8}\left(\frac{m}{2kT}\right)^{-5/2} = \boxed{\frac{3kT}{m}}$$

$$\langle \varepsilon \rangle = \frac{1}{2}m\langle v^2 \rangle = \underbrace{3}_{\text{D.O.F.}} \cdot \frac{1}{2}kT \qquad \boxed{\text{equipartition theorem}}$$

Useful integrals

$$\int_{-\infty}^{\infty} e^{-x^2}dx \cdot \int_{-\infty}^{\infty} e^{-y^2}dy = \int_0^\infty e^{-r^2} r\,dr \int_0^{2\pi} d\theta = \pi \int_0^\infty e^{-u}du = \pi$$

thus: $\displaystyle\int_0^\infty e^{-\alpha v^2}dv = \frac{\sqrt{\pi}}{2}\alpha^{-1/2}$ — $\displaystyle\int_0^\infty v e^{-\alpha v^2}dv = \frac{1}{2\alpha}$

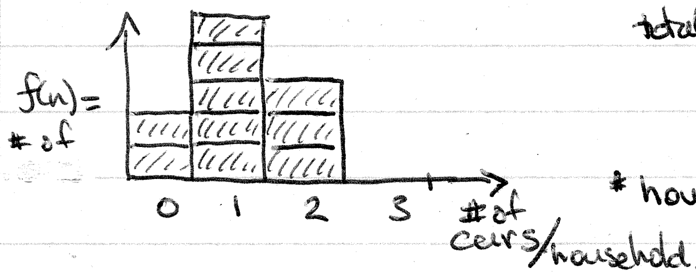
$\frac{d}{d\alpha}$: $\displaystyle -\int_0^\infty v^2 e^{-\alpha v^2}dv = \frac{-\sqrt{\pi}}{4}\alpha^{-3/2}$ — $\displaystyle -\int_0^\infty v^3 e^{-\alpha v^2}dv = \frac{-1}{2\alpha^2}$

$\frac{d^2}{d\alpha^2}$: $\displaystyle\int_0^\infty v^4 e^{-\alpha v^2}dv = \frac{3\sqrt{\pi}}{8}\alpha^{-5/2}$ — $\displaystyle\int_0^\infty v^5 e^{-\alpha v^2}dv = \frac{1}{4\alpha^3}$

Distributions

* things you "add up" or integrate.
* AKA: density, differential form, measure, weight, PDF, histogram (prob. dist. function)
* normalization, moments.

1) how many cars per household?



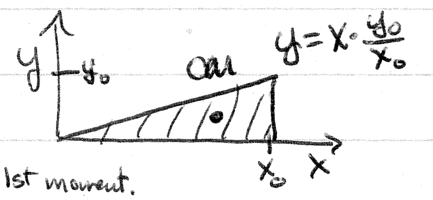
$$\text{total \# cars} = \Sigma n \cdot f(n) \quad \text{(1st moment)}$$
$$= 0+0 + 1+1+1+1+1 + 2+2+2$$
$$= 0\cdot 2 + 1 \cdot 5 + 2 \cdot 3 = 11$$

$$\text{* households} = \Sigma f(n) = 2+5+3 = 10 \quad \text{(0}^{th}\text{ moment)}$$

* average, expectation value, 1st moment:

$$\bar{n} = \langle n \rangle = \frac{\sum_n n \cdot f(n)}{\sum_n f(n)} = \frac{11}{10} = 1.1$$

2) centre of mass:



$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^{x_0} x \cdot \rho_0 \left(\frac{y_0}{x_0} x\right) dx}{\int_0^{x_0} \rho_0 \left(\frac{y_0 x}{x_0}\right) dx}$$

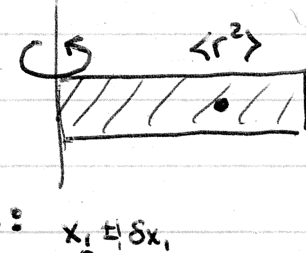
1st moment.

mass = 0th moment

$$= \frac{\int_0^{x_0} x^2 dx}{\int_0^{x_0} x\,dx} = \frac{x^3/3\,\big|_0^{x_0}}{x^2/2\,\big|_0^{x_0}} = \frac{2}{3} x_0$$

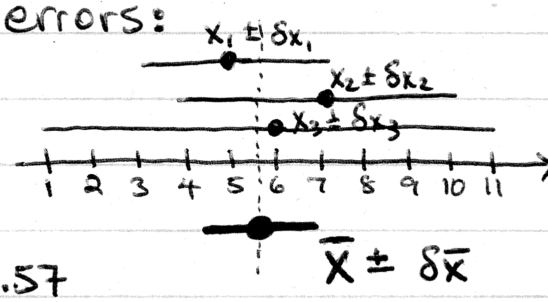
$$y_{cm} = 2/3\, y_0$$

3) moment of inertia (2nd moment)



$$I = \int r^2 dm = \langle r^2 \rangle m$$

4) weighted average w/ statistical errors:



$$\bar{x} = \langle x \rangle_\omega = \frac{\omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3}{\omega_1 + \omega_2 + \omega_3}$$

note: $\delta x \sim \frac{1}{\sqrt{n}}$ so $\omega \sim n \sim \frac{1}{\delta x^2}$

$$\bar{x} = \frac{\omega \bar{x}}{\omega} = \frac{\frac{1}{4}\cdot 5 + \frac{1}{9}\cdot 7 + \frac{1}{25}\cdot 6}{\frac{1}{4} + 1/9 + 1/25} = 5.65 \pm 1.57$$