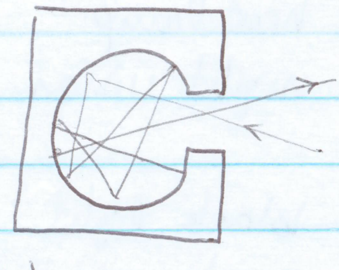
blackbody radiation



blackbody - absorbs all incident radiation

- thermal equilibrium: re-radiates
- nonexamples: every-day color - reflection

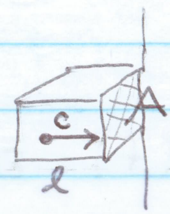
Stefan-Boltzman law: $R = \sigma T^4$ $\qquad \sigma = 5.67\times10^{-8} \frac{W}{m^2 K^4}$

$R = \text{intensity} = \frac{\text{power}}{\text{area}} = \frac{c}{4} u$

$u = \text{energy density} = \frac{\text{energy}}{\text{volume}}$

$= \int u(\lambda) d\lambda$ spectral density

$= \int u(\nu) d\nu$



Wien's Displacement law:

$\lambda_{max} \cdot T = 2.898\times10^{-3}\, m\cdot K$

color $\Leftrightarrow$ temperature

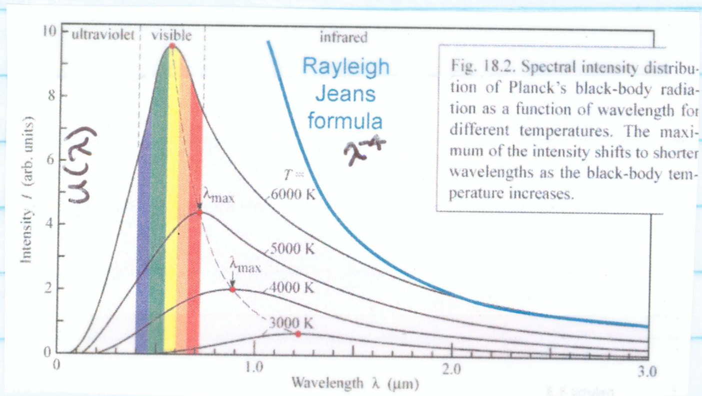


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

Rayleigh-Jeans equation:

$u(\nu) = \underbrace{G(\nu)}_{\text{density of states}} \cdot \underbrace{\bar{\varepsilon}(\nu)}_{\text{average energy per state}}$

$$\boxed{u(\nu) = \frac{8\pi\nu^2}{c^3} \cdot kT}$$

density of states: use $L = \frac{n\lambda}{2} = \frac{nc}{2\nu}$

half wavelength.

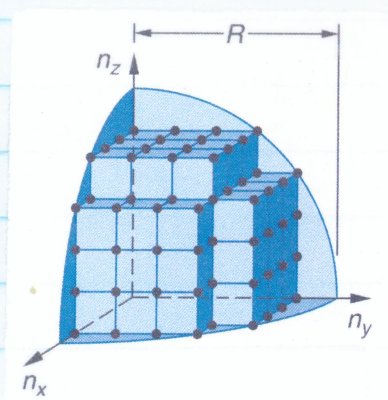
1D: $dn = \frac{2L}{c} d\nu = L \cdot G(\nu) d\nu$

two polarizations

3-D: $G(\nu)d\nu = \frac{2}{8} \frac{d^3 n}{V} = \frac{2}{8} \frac{4\pi n^2 dn}{L^3}$

one octant

$= \frac{8\pi}{c^3} \nu^2 d\nu$



$d^3 n = 4\pi n^2 dn$

average energy:

equipartition theorem: $\bar{\varepsilon}(\nu) = 2 \cdot \frac{1}{2} kT$

two degrees of freedom

kinetic + potential energy.

Planck's law: $u(\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1}$ • at first, just a fit to the data

as $\nu \to 0$ $e^x \to 1 + x + x^2/2 + \ldots$

so $u(\nu) \to \frac{8\pi\nu^2}{c^3} kT$ as in Rayleigh-Jeans formula.

as $\nu \to \infty$ $u(\nu) \to 0$

Classical:

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon e^{-\varepsilon/kT} d\varepsilon}{\underbrace{\int_0^\infty e^{-\varepsilon/kT} d\varepsilon}} = \frac{\left(\frac{1}{kT}\right)^{-2}}{\left(\frac{1}{kT}\right)^{-1}} = \boxed{kT}$$

$$Z = \int_0^\infty e^{-\alpha\varepsilon} d\varepsilon = \frac{1}{\alpha} \quad \text{(bottom)}$$

$$\frac{dZ}{d\alpha} = -\int_0^\infty \varepsilon e^{-\alpha\varepsilon} d\varepsilon = \frac{-1}{\alpha^2} \quad \text{(top)}$$

Planck: let $\varepsilon_n = nh\nu$

$$\int_0^\infty d\varepsilon \to \sum_{n=0}^\infty$$

- oscillator emits energy in "quanta"
- higher frequency in larger energy packets.

$$\bar{\varepsilon} = \frac{\sum_{n=0}^\infty \varepsilon_n e^{-\varepsilon_n/kT}}{\underbrace{\sum_{n=0}^\infty e^{-\varepsilon_n/kT}}} = \frac{e^x \cdot h\nu}{1-e^x} = \boxed{\frac{h\nu}{e^{h\nu/kT} - 1}}$$

$$Z = \sum_{n=0}^\infty (e^x)^n = \frac{1}{1-e^x}$$

where $x = \frac{-h\nu}{kT}$

$$\frac{dZ}{dx} = \sum_{n=0}^\infty n(e^x)^n = \frac{e^x}{(1-e^x)^2}$$