Matter Waves

- de Broglie wavelength.

$\hbar = \frac{h}{2\pi}$ $\quad \hbar c = 197\,\text{eV}\cdot\text{nm}$

E	f
p	λ

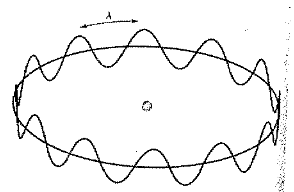
$E = hf = \hbar\omega$ $\quad \omega = 2\pi f$ = "angular frequency"

$p = h/\lambda = \hbar k$ $\quad k = \frac{2\pi}{\lambda}$ = wave number (spatial ang. frequency)

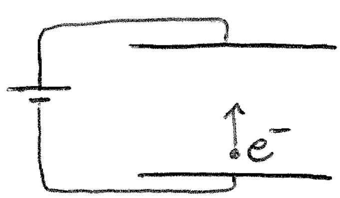
- interpretation of Bohr quantization condition

$$L = n\hbar$$
$$rp = n\frac{h}{2\pi}$$
$$2\pi r = n\lambda$$

FIGURE 6.1
If an electron wave — whatever it may be — is pictured as circling around the atomic nucleus, its wavelength λ must fit an integer number of times into the circumference.



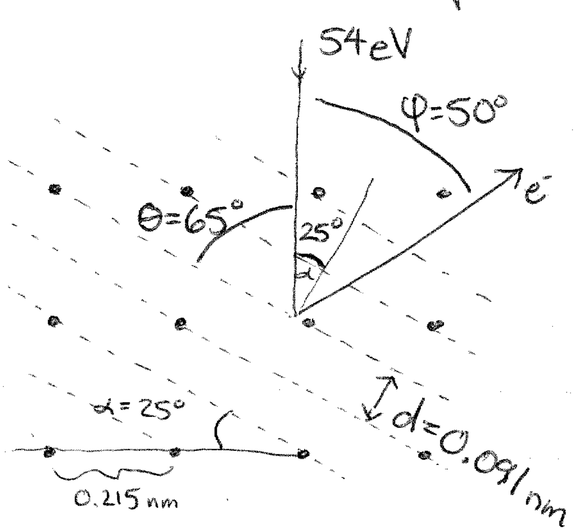
- example: wavelength of 54eV electron



$$E = \tfrac{1}{2}mv^2 = 54\,\text{eV}$$
$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\cdot 54\,\text{eV}}{0.511\times10^{6}\,\text{eV}/c^2}} = 0.0145c = 4.4\times10^{6}\,\text{m/s}$$

$$p = mv = 0.511\,\text{MeV}/c^2 \cdot 0.0145c = 7.42\,\text{keV}/c$$
$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\,\text{eV}\cdot\text{nm}}{7420\,\text{eV}} = \boxed{0.167\,\text{nm}}$$

- Davisson-Germer experiment



$$n\lambda = 2d\sin\theta$$
$$= 2\cdot(0.215\,\text{nm}\cdot\sin 25°)\cdot\sin(65°)$$
$$= 2\cdot 0.091\,\text{nm}\cdot\sin 65°$$
$$= \boxed{0.165\,\text{nm}}$$

Bragg scattering of an electron!

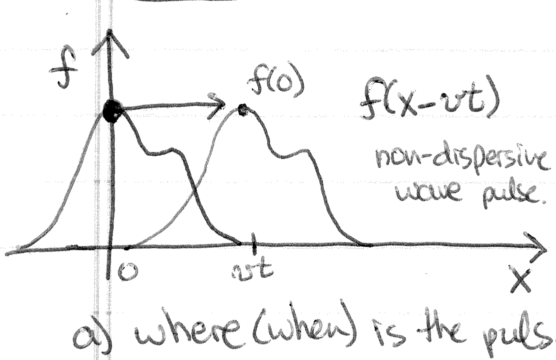
- neutron scattering
- electron microscopes

	property	PARTICLE	(duality)	WAVE
how they behave	interaction	collision		interference (superposition)
	propagation	ballistic motion		diffraction (Huygen's principle)
		scatter/bounce/absorb		reflection / refraction
classical description	state	trajectory $\vec{x}(t)$		wavefunction $E(\vec{x},t)$
	parameters	$\vec{x}, t$		λ, ν or k, ω
	kinematics	$\vec{v} = \frac{d\vec{x}}{dt}$		$v = \lambda\nu = \frac{d\omega}{dk}$
	dynamics	$\vec{F} = m\vec{a}$		$\frac{\partial^2 E}{\partial t^2} = v^2 \frac{\partial^2 E}{\partial x^2}$ wave eq'n.
	conservation	$E = \frac{1}{2}mv^2$ $p = mv$		amplitude, oscillation $T \leftrightarrow V$
	inertia	m		dispersion $\omega(k)$
	degeneracy	[spin]		polarization
	(quantization)	m, e		modes (boundary conditions)
quantum description	quantization of E, p	matter waves, wavefunction modes	$p = h/\lambda$ ⤫ $E = h\nu$	photons, quantum of energy
	interpretation	probability amplitude		wave packet
	duality	Born interpretation		Heisenberg uncertainty principle
	manifestation	INTERACT as particles (observation)		PROPAGATE as waves

what are light & matter?

wave	medium	oscillation	intensity	particle
light	$\vec{E}, \vec{B}$	$E \leftrightarrow B$	$\vec{S} = \vec{E} \times \vec{B} \propto E^2$	photon γ
matter	Ψ	$Re\Psi \leftrightarrow Im\Psi$	$\lvert\Psi\rvert^2$	*on $e, p, n, \ldots$

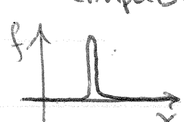
Wavefunctions



$f(x - vt)$

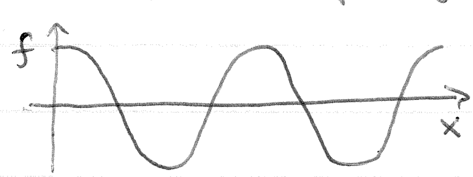
non-dispersive wave pulse.

a) localized pulse (impulse)



$\delta(x - x_0)$

b) pure frequency



$$f = A\cos\left(2\pi\frac{x}{\lambda} - 2\pi\frac{t}{T}\right)$$

($2\pi\frac{t}{T}$: νt) $\qquad x - \left(\frac{\lambda}{T}\right)t$ ($\frac{\lambda}{T} = v$) $\qquad x - (\lambda\nu)t$

$$= A\cos(k \cdot x - \omega t) \qquad x - \left(\frac{\omega}{k}\right)t$$

$k = \frac{2\pi}{\lambda}$ wavenumber $\qquad \omega$ = angular freq.

a) where (when) is the pulse?

b) what is the frequency/wavelength?