## University of Kentucky, Physics 404G Exam 1, Friday, 2020-10-02

Instructions: This take-home exam is due on Sunday 2020-10-04 at 11:59 pm. You may not consult with any other source besides the course textbooks and the lectures and notes posted on the course website. In-person or online help is strictly prohibited, including searches on the internet or online homework solution sites. Show intermediate work for partial credit.

1. A pointlike person of mass m sits at the end of a rope of length d attached to a trolley, which slides along a zipline with the profile  $y(x) = x^2$  (the height y of the cable as a function of horizontal position x). Assume that all zipline components are massless, that the height profile of the cable does not change as the trolley slides along it, and that the rope is always taut (straight line).

[10 pts] **a)** Calculate the Christoffel symbol  $\Gamma_{\psi\xi\psi}$  using the generalized coordinates  $(\xi, \psi)$  of the person, where  $\xi$  is the *x*-coordinate of the trolley along the zipline, and  $\psi$  the angle of the rope with respect to vertical.

[10 pts] **b**) Calculate the 2nd order equations of motion in  $(\xi, \psi, \dot{\xi}, \dot{\psi})$  using the Lagrangian (you do not need to solve the differential equations).

[5 pts] c) [bonus: Calculate the equations of motion in  $(p_{\xi}, p_{\psi}, \xi, \psi)$  using the Hamiltonian.]



2. A positron (positive electron) starts at rest at the origin at t = 0 in an electromagnetic spectrometer with constant fields  $\vec{B} = \hat{z}B_0$  and  $\vec{E} = \hat{x}E_0$ , where  $B_0 = 10 \ \mu\text{T}$  and  $E_0 = 10 \ \text{V/m}$ .

[10 pts] a) Calculate the position of the electron at  $t = 1 \mu s$ , either analytically or numerically.

[10 pts] **b**) Calculate  $(E_1, B_1)$  after the first iteration of the Newton-Raphson method to tune the final position at  $t = 1 \ \mu$ s to (x, y) = (50, -50) cm by adjusting the magnitude of the electric and magnetic fields starting from the fields  $(E_0, B_0)$ .