**University of Kentucky, Physics 404G**
**Final Exam, 2020-11-27**

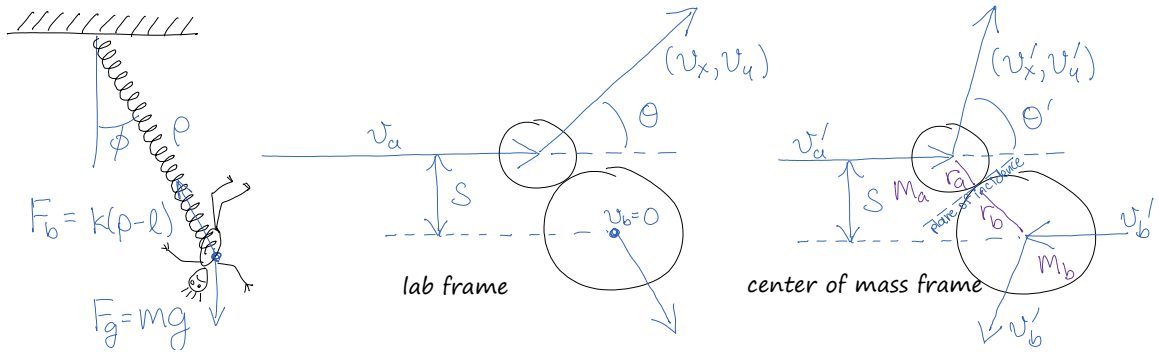
Instructions: This take-home exam is due on Sunday 2020-11-29 at 11:59 pm. You may not consult with any other source besides the course textbooks and the lectures and notes posted on the course website. In-person or online help is strictly prohibited, including searches on the internet or online homework solution sites. Show intermediate work for partial credit.

## 1. Pretzel-bungee-jumper

[10 pts] **a)** Write the Lagrangian in circular coordinates $(\rho, \phi)$ of a person of mass $m$ hanging under the weight $mg$ of gravity from a bungee cord, approximated as a spring of natual length $\ell$ and spring constant $k$.

[10 pts] **b)** Use Lagrange's equations to calculate the equations of motion $\ddot{\rho}(\rho, \phi, \dot{\rho}, \dot{\phi})$ and $\ddot{\phi}(\rho, \phi, \dot{\rho}, \dot{\phi})$, and identify the Coriolis and centrifugal terms.

[ 5 pts] **c)** Integrate the equations of motion numerically and plot the trajectory $(x(t), y(t))$ for the first two minutes, given $m$=100 kg, $\ell$=10 m, $k$=100 N/m, $\rho_0$=1 m, $\phi_0$=10°, $\dot{\rho}_0$=1 m/s, $\dot{\phi}_0$=0.




**2. Hard scattering with recoil**–A projectile ball of mass $m_a$ and radius $r_a$ is thrown with velocity $\hat{\boldsymbol{x}} v_a$ and impact parameter $s$, bouncing elastically off of a stationary ball ($v_b = 0$) of mass $m_b$ and radius $r_b$, which recoils backward from the collision.

[ 5 pts] **a)** Calculate the initial velocity $\hat{\boldsymbol{x}} v'_a$ of the projectile in the center of mass reference frame, in which the total momentum equals zero.

[ 5 pts] **b)** Calculate the final velocity $(v'_x, v'_y)$ of the projectile in the center of mass frame. Using conservation of energy and momentum and assuming an impulse perpendicular to the surface of impact, the ball 'reflects' from the plane of incidence, as it does in Taylor Example 14.5. Note that the plane of incidence is stationary in the center of mass frame.

[ 5 pts] **c)** Calculate the final velocity $(v_x, v_y)$ and scattering angle $\theta$ of the projectile in the laboratory frame.

[ 5 pts] **d)** Calculate the scattering cross section $d\sigma/d\Omega$ for this interaction.

[ 5 pts] **e)** What is the total cross section?