University of Kentucky, Physics 404G
Homework #9, Rev. A, due Tuesday, 2021-11-23

1. Chalkboard scattering Perform a scattering experiment to measure the cross-sectional area σ of each of T identical targets drawn randomly on a chalkboard. Throw as many racquetballs as you can and record the number H of hits versus M of misses. What other measurements do you need to extract the total cross section σ of the target? Calculate σ and its statistical uncertainty from your data and compare to the actual area of the target. Note that the cross section includes information about both the target and the scatterer—what area are we actually measuring experimentally? How could you generalize this experiment to measure a differential cross section?

2. Rutherford scattering of α particles ($Z = 2$) from the atoms ($Z = 79$) in a thin foil of gold due is due to the Coulomb repulsion $F = Z_1 Z_2 e^2/4\pi\epsilon_0 r^2$ of the nuclei in the target. This inverse square law has the same form as gravity, with the same trajectories as the unbound Kepler problem (H08 #3, with positive total energy $E > 0$). The repulsive potential corresponds to the opposite branch of the hyperbola than the attractive potential.

a) Calculate the Rutherford cross section $d\sigma/d\Omega$ as a function of scattering angle θ and energy E of an α-particle scattering from a gold nucleus. You may assume that the α particle is energetic enough that electron screening is negligible.

b) What is the probability that an α particle would reflect within 1° of straight backwards from a 1 μm thick gold foil?

3. A **rainbow** is formed by rays of sunlight refracting through raindrops, internally reflecting, and refracting back out toward the viewer.

a) Calculate the scattering angle θ as a function of the impact parameter s for a given index of refraction n of the raindrop. Calculate the rainbow angle θ_0 as the *caustic* where $d\theta/ds|_{\theta_0} = 0$. Much of the incoming light scatters in this direction because of the zero derivative.

b) Calculate the cross section $d\sigma/d\Omega$ as a function of s (not *theta*!). Show that the peak in the cross section at $s = s_0$ occurs at the same angle $\theta_0 = \theta(s_0)$ as part a). Explain why the inside of a rainbow is brigher than the outside.

c) [bonus: Calculate the scattering angle θ for m internal reflections (eg. double rainbow).]

4. Optical potential

The *optical potential* $V(r) = V_0\,\theta(a-r)$ is the mechanical analog of rainbow scattering. It describes particles that slow down [or speed up] as they enter a spherical target and return to their original speed as they exit the other side. The constant potential V_0 accounts for the loss [gain] of kinetic energy inside the sphere of radius a. The gradient $\delta(a-r)$ of the step function $\theta(a-r)$ represents a finite impulse at the surface of the sphere due short range repulsion/attraction from material inside the target. This causes a deflection from Snell's on the particle's wave function just the same as the optical effect.

a) What is the 'index of refraction' $n = v/v_0$ of a particle in terms ratio E/V_0 of total energy to potential well height/depth for a particle of mass m?

b) Calculate the scattering cross section $d\sigma/d\Omega$ as a function of s assuming no internal reflections, using the method of problem #3.