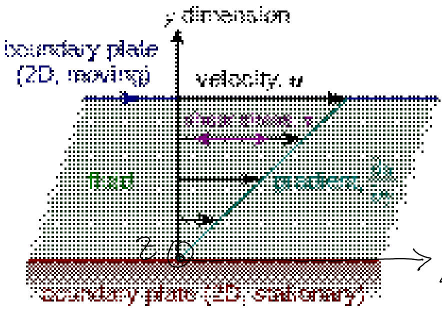
L04 Damped motion

Monday, September 24, 2018 04:01

- *Linear Drag: Viscosity – https://en.wikipedia.org/wiki/Viscosity*
  See also: PPT Purcell, "Life at Low Reynolds Number"
  and https://en.wikipedia.org/wiki/Pitch_drop_experiment



"coefficient of friction in a fluid"

$$\underbrace{\tau_{xy}}_{\text{shear stress}} \cong F_x / A_y = F_x / dx\,dz$$

$$= \underbrace{\eta}_{\text{viscosity}} \frac{\Delta u}{\Delta y}$$

Taylor ch2 #2.2.

STP: $\eta = 1.7 \times 10^{-5}$ N·s/m^2

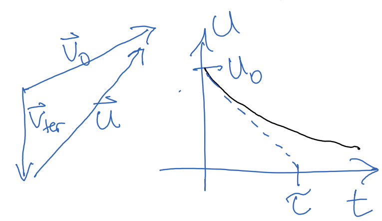
$\beta = 1.6 \times 10^{-4}$ N·s/m^2

See Taylor, footnote 8, p72: $F = \eta A\, dv/dy$ $\qquad f_{\text{lin}} = \underbrace{3\pi\eta D}_{b} v$ *(sphere)*

* 3d solution to linear damping in gravity:

$$\vec{F} = m\dot{\vec{v}} = \underbrace{m\vec{g}}_{b\vec{v}_{ter}} - b\vec{v} \qquad \text{let } \vec{u} = \vec{v} - \vec{v}_{ter} \qquad \dot{\vec{u}} = \dot{\vec{v}} \qquad b\vec{v}_{ter} = m\vec{g}$$

$$m\dot{\vec{u}} = -b\vec{u} \qquad \text{let } \tau = \frac{m}{b} \qquad \vec{u} = \hat{u}\,u$$



$$\int_{u_0}^{u} \frac{du}{u} = \int_0^t -\frac{dt}{\tau} \qquad \ln u/u_0 = -t/\tau \qquad u = u_0 e^{-t/\tau}$$

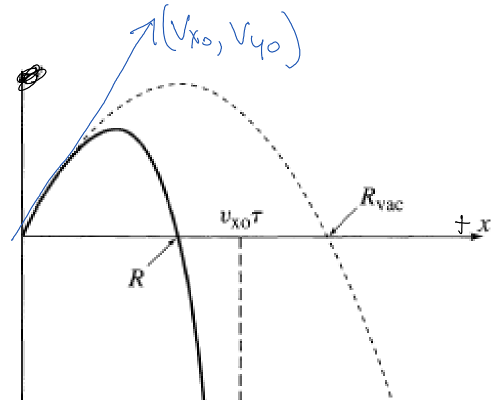
$$(\vec{v} - \vec{v}_{ter}) = (\vec{v}_0 - \vec{v}_{ter})\, e^{-t/\tau} \qquad \vec{v}_{ter} = \frac{m\vec{g}}{b} = \tau\vec{g} \qquad \tau = m/b$$

$$\vec{r}(t) = \vec{r}_0 + \int_{t=0}^{t} \vec{v}(t) = \boxed{\vec{r}_0 + \vec{v}_{ter}t - (\vec{v}_0 - \vec{v}_{ter})\,\tau\,(e^{-t/\tau} - 1)}$$

- trajectory: let $\vec{r}_0 = (0,0)$ $\quad \vec{v}_0 = (v_{x0}, v_{y0})$, note $\vec{v}_{ter} = (0, v_{ter})$

$$v_x = v_{x0} e^{-t/\tau} \qquad x = \cancel{x_0} + \int_0^t v_x dt = v_{x0}(-\tau)e^{-t/\tau}\Big|_0^t = v_{x0}\tau(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = \frac{x}{v_{x0}\tau} \qquad t = -\tau \ln\left(1 - \frac{x}{v_0\tau}\right)$$



$$v_y = v_{ter} + (v_{y0} - v_{ter})\, e^{-t/\tau} \qquad [+ \text{ going down}]$$

$$y = \int_0^t v_y dt = \cancel{y_0} + v_{ter}t + (v_{y0} - v_{ter})\,\tau\,(1 - e^{-t/\tau})$$

$$= v_{ter}\left(-\tau \ln\left(1 - \frac{x}{v_{x0}\tau}\right)\right) + (v_{y0} - v_{ter})\cancel{\tau}\left(\frac{x}{v_{x0}\cancel{\tau}}\right)$$

$$= v_{ter}\left(-\tau \ln\left(1-\frac{x}{v_{x_0}\tau}\right)\right) + (v_{y_0}-v_{ter})\tau\left(\frac{x}{v_{x_0}\tau}\right)$$

$$= \frac{v_{y_0}-v_{ter}}{v_{x_0}}\, x \;-\; v_{ter}\tau \ln\left(1-\frac{x}{v_{x_0}\tau}\right)$$

$+y \downarrow$ [down]

$\xrightarrow{x\approx 0} \dfrac{v_{y_0}x}{v_{x_0}}$, $\quad \xrightarrow{x\approx v_{x_0}\tau} -\infty$

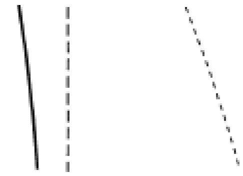


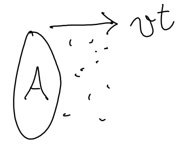
fig 2.7 p55, Taylor

- *Quadratic drag: the force required to accelerate the air in front to v*

$dm = \rho\, dV = \rho A v\, dt$

$\kappa = 1/4$ "streamlined" $\quad \gamma = 0.25\ \mathrm{N{\cdot}s^2/m^4}$

$\varrho = 1.29\ \mathrm{kg/m^3}$ STP



$F = \left(ma = dm\,\frac{\Delta v}{dt}\right)\cdot \kappa$ fudge factor

$$f_{quad} = \kappa \varrho A v^2 = \underbrace{\kappa \rho \tfrac{\pi}{4} D^2}_{c}\, v^2$$

(Taylor ch2 #2.4)

* 1-d solution for quadratic drag

$F = \underbrace{mg}_{c v_{ter}^2} - cv^2 = m\underbrace{\dot{v}}_{dv/dt}$ $\qquad$ let $\quad mg = c v_{ter}^2$

$\dot{v} = g\left(1 - v^2/v_{ter}^2\right)$

$\dfrac{dv}{1 - v^2/v_{ter}^2} = g\,dt$ $\qquad$ let $\quad v = v_{ter}\cdot \tanh \alpha$

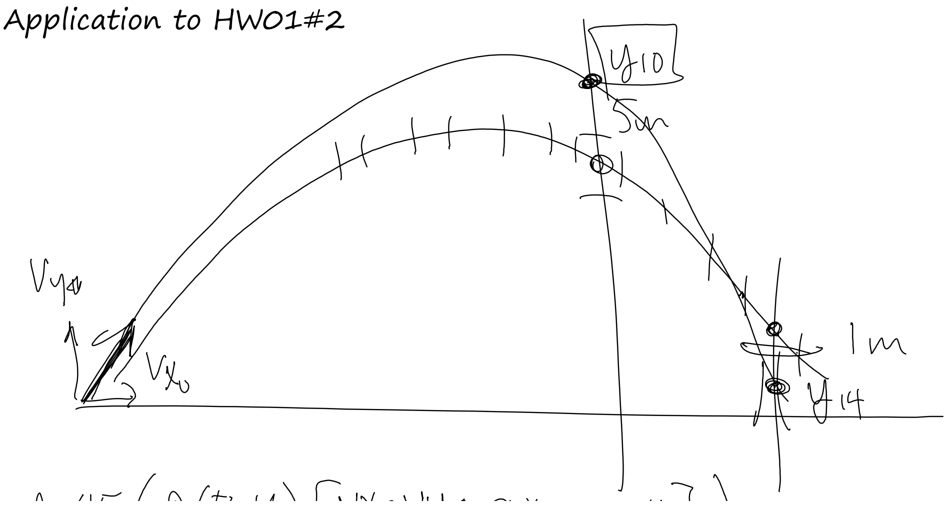
$\cosh^2\alpha - \sinh^2\alpha = 1$

$v_{ter}\,\dfrac{\mathrm{sech}^2\alpha\, d\alpha}{\mathrm{sech}^2\alpha} = g\,dt$ $\qquad$ $1 - \tanh^2\alpha = \mathrm{sech}^2\alpha$

$v_{ter}\,\alpha = g t$ $\qquad$ $dv = v_{ter}\cdot \mathrm{sech}^2\alpha \cdot d\alpha$

$v = v_{ter} \tanh\left(gt/v_{ter}\right)$

- *Application to HW01#2*



ode45(@(t,u) [vx; vy; gx , gy])

$u(3) \quad u(4)$

$$\vec{F} = m\vec{a} = m\vec{g} - cv^2\hat{v}$$

$$u = \begin{bmatrix} x \\ y \\ vx \\ vy \end{bmatrix} \qquad \vec{a} = \vec{g} - cv\vec{v} \qquad \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{v^2} = v$$

$$\vec{a} = \vec{g} - c\sqrt{\vec{v}\cdot\vec{v}}\,\vec{v}$$

$$a_x = g_x - c\sqrt{v\cdot v}\; v_x$$

$$= g_x - c \text{ sqrt(dot(u(3:4), u(3:4)))} * u(3)$$