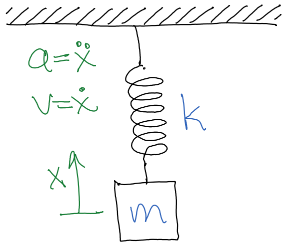
L17 Oscillators

Thursday, September 24, 2020 12:07



note: mg absorbed in equilibrium $x_0=0$.

$$F = ma = -kx \qquad \ddot{x} + \omega_0^2 x = 0 \qquad \omega_0^2 = k/m$$

$$\text{let } x = e^{st} \qquad s^2 x + \omega_0^2 x = 0 \qquad s = \pm i\omega_0$$

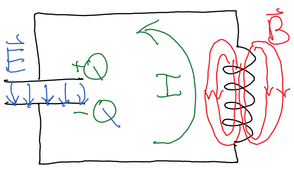
$$x = C_1 \underbrace{e^{i\omega_0 t}}_{\cos\omega_0 t + i\sin\omega_0 t} + C_2 e^{-i\omega_0 t} = \underbrace{B_1}_{C_1+C_2}\cos\omega_0 t + \underbrace{B_2}_{i(C_1-C_2)}\sin\omega_0 t$$

$$2C_1 = B_1 - iB_2 = C_2^*$$

$$= \mathrm{Re}\{\underbrace{C}_{Ae^{-i\delta}} e^{i\omega_0 t}\} = A\cos(\omega_0 t - \delta) = \underbrace{A\cos\delta}_{B_1}\cos\omega_0 t + \underbrace{A\sin\delta}_{B_2}\sin\omega_0 t$$

$$\mathcal{H} = T + V = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \text{const.} \qquad T_{max} = V_{max}$$

$$= \tfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t - \delta) + \tfrac{1}{2}kA^2\cos^2(\omega t - \delta) =$$

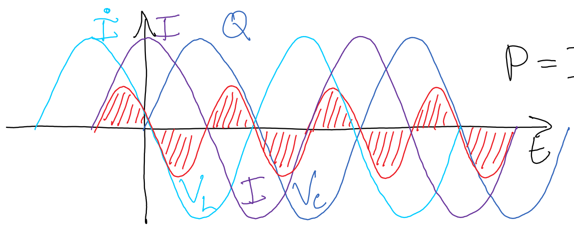


$$Q = CV \qquad V = L\dot{I}$$

[Coulomb] [Faraday]

$$L\ddot{Q} + \tfrac{1}{C}Q = 0$$

$$\ddot{Q} + \omega_0^2 Q = 0 \qquad \omega_0^2 = \frac{1}{LC}$$



$$P = IV = \underbrace{Q_0\omega\cos(\omega_0 t - \delta)\cdot\frac{Q_0}{C}\sin(\omega_0 t - \delta)}_{\text{[capacitor]}} = \frac{Q_0^2\omega_0}{2C}\sin 2(\omega_0 t - \delta)$$

$$= \underbrace{I_0\sin(\omega_0 t - \delta)\cdot L(-I_0\omega_0)\cos(\omega_0 t - \delta)}_{\text{[inductor]}} = LI_0^2\omega_0\sin(2\omega_0 t - \delta)$$