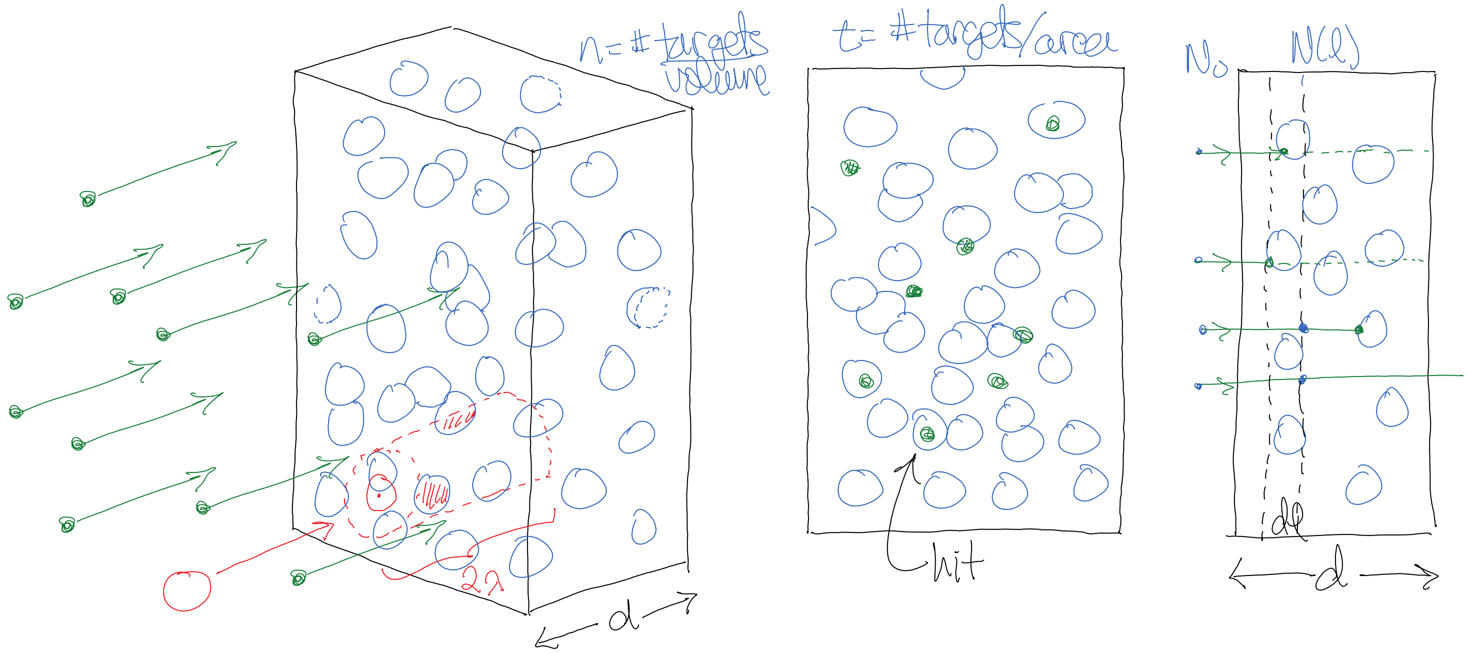
L33 Scattering cross section

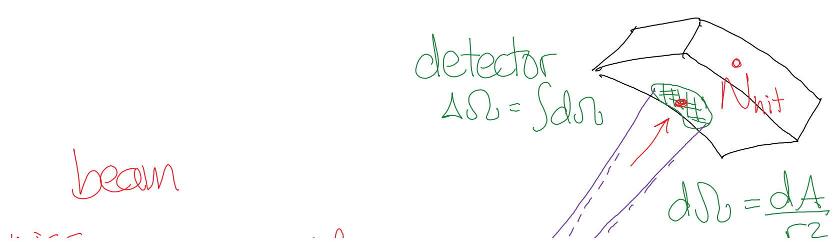
Tuesday, November 3, 2020 18:09

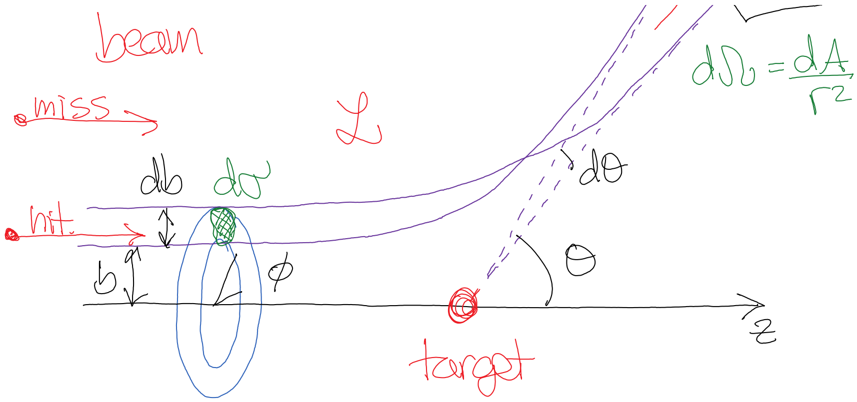
- To first order, the scattering of a beam of particles from the atoms in a material target can be described geometrically.
  - If we flatten the target to 2-d, the probability of hitting a target is proportional to the "cross sectional area", which we just call "cross section" of the interaction.
  - Even in quantum mechanics the cross section σ is defined to behave in exactly this way
  - The "target thickness" $t = nd = N_{targ}/A$ is the areal density of targets viewed head-on, where target density $n = N_{targ}/V$
  - Scattering cross sections are measured by counting the interactions, divided by the luminosity $\mathcal{L} = \dot{N}_{beam} \cdot t$
- The mean-free path λ is defined as the average distance traveled before hitting a target, ie. $n\sigma\lambda = 1$
  - As the beam attenuates exponentially as it passes through a thick target ($d \gg \lambda$)

$$dN = -N n\sigma \, dl \qquad \frac{dN}{N} = d \ln N = -n\sigma dl \qquad N(l) = N_0 e^{-nl\sigma}$$



- Usually we detect interactions by deflection of the beam
  - This gives us extra information: the scattering angle θ , a function of the impact paramter b (also called s)
  - Most problems are azimuthally symmetric (independent of ϕ) (the beam is directed along $+z$)
  - We break σ down into small pieces $d\sigma$ 'differential cross section' for each direction scattered into
  - All beam particles intercepted by $d\sigma$ scatter into a cone with solid angle $d\Omega = dA/r^2$.
  - The ratio $d\sigma/d\Omega$, what is actually called the differential cross section is measured in nuclear/atomic expts.
  - It can be calculated theoretically from the relation $\theta(b)$ which comes from the dynamics of the interaction
  - The 'total cross section' is $\sigma_t = \int d\sigma = \int_{4\pi} d\Omega \, {}^{d\sigma}/_{d\Omega}$





$$\frac{d\sigma}{d\Omega} = \frac{\dot{N}_{hit}}{\mathcal{L}\,\Delta\Omega} \quad \text{[expt]}$$

$$= \frac{b\,db\,\cancel{d\phi}}{s_\theta\,d\theta\,\cancel{d\phi}} \quad \text{[theory]}$$