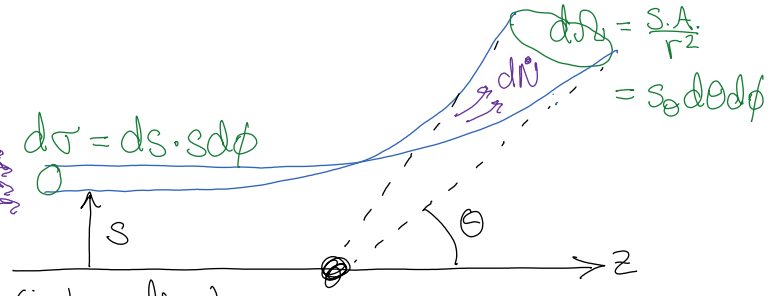
L34 Rutherford scattering

Tuesday, November 3, 2020 20:49

- Review of differential cross section

$$\frac{d\dot{N}}{\mathcal{L}\,d\Omega} = \frac{d\sigma}{d\Omega}(\theta) = \left|\frac{s\,ds}{s_\theta\,d\theta}\right|$$



$d\Omega$ = acceptance (solid angle)

$d\sigma$ = effective cross-section area (interaction)

$\mathcal{L} = I \cdot t$ = beam current [#/time] × target thickness [#/area]

$\dot{N} = \mathcal{L} \cdot d\sigma$ = [#/area·time] × [area] = #/time count rate

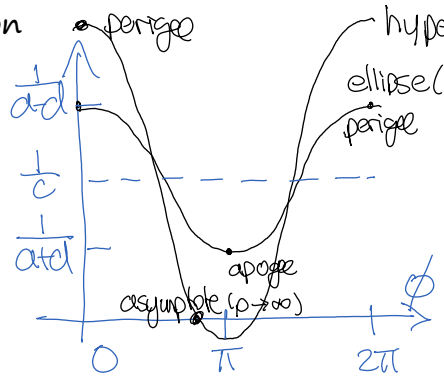
- Review of inverse-square law solution

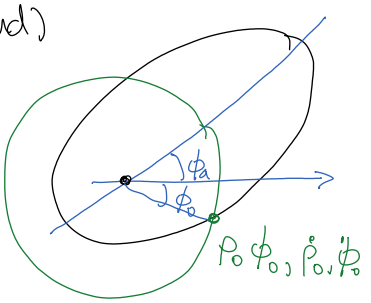
$$\frac{1}{p} = u = \frac{1}{c}\left(1 + e\cos(\phi - \phi_a)\right)$$

$$c = \frac{m\gamma}{l^2} \qquad e^2 = 1 + \frac{2El^2}{m\gamma^2}$$

$$l = m\rho^2\dot{\phi} \qquad E = \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\phi}^2) - \frac{\gamma}{\rho}$$

$\rho_0, \phi_0, \dot{\rho}_0\dot{\phi}_0 \rightarrow l, E \rightarrow c, e \rightarrow \phi_a - \phi_0$





- Rutherford scattering cross section

$$l^2 = (mvs)^2 = 2mE \cdot s^2 \qquad e^2 = 1 + 4E^2s^2/\gamma^2$$

$$1 + \sqrt{1 + 4E^2s^2/\gamma^2}\cos\phi_0 = 0 \qquad 2\phi_0 + \theta = \pi$$

$$\sin\frac{\theta}{2} = \frac{-1}{\sqrt{1 + 4Es^2/\gamma^2}} \qquad \sin\frac{\theta}{2} = \cos\phi_0$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\frac{1}{2}\cos\frac{\theta}{2}\,d\theta = \frac{1}{2}\left(1 + 4s^2E^2/\gamma^2\right)^{-3/2} \cdot 8E^2 s\,ds/\gamma^2$$

$$\sin\theta\,d\theta = -2\sin^4\frac{\theta}{2} \cdot 8E^2 s\,ds/\gamma^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left|\frac{s\,ds}{s_\theta\,d\theta}\right| = \frac{\gamma^2}{16E^2}\sin^{-4}\frac{\theta}{2}}$$