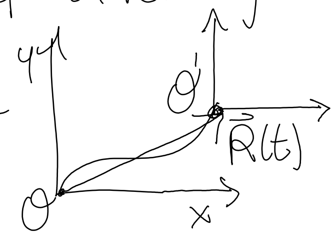
L38 Inertial Forces

Wednesday, December 4, 2019 07:30

* Galilean relativity – the foundation behind special relativity

let $Oxyz$ be a inertial frame where $\vec{F} = m\vec{a}$, and let $O'xyz'$ be a frame with the same unit vector $\hat{x}, \hat{y}, \hat{z}$ but O' is translated to $\vec{R}(t)$ w/r O.



then $\vec{r}' = \vec{r} - \vec{R}$ $\quad \vec{v}' = \vec{v} - \underbrace{\dot{\vec{R}}}_{\vec{V}}$ $\quad \vec{a}' = \vec{a} - \underbrace{\ddot{\vec{R}}}_{\vec{A}}$

or $m\vec{a}' = \vec{F} - m\vec{A}$

the acceleration of O' ie $\vec{A}$ is called an inertial force $\vec{F}_{inertial} = -m\vec{A}$ and acts just like a real force

that is the general equivalence principle behind general relativity.

* Rotating frames: we have already seen that $[\vec{\omega}\times]$ generates rotations about the $\vec{\omega}$ axis. Here's a simpler way to see this:

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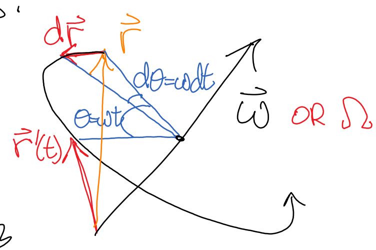
$d\vec{s} = \vec{\omega} \times \vec{r}$ $\qquad \frac{d}{dt}\vec{r} = \vec{\omega} \times \vec{r}$



this holds for all vectors: $\hat{e}_1, \hat{e}_2, \hat{e}_3$

let $Ox'y't'$ be rotating at velocity $\vec{\omega}(t)$ about $Oxyz$, with the same origin

then $\vec{v} = \frac{d}{dt}\hat{e}_i r'^i = \hat{e}_i \dot{r}'^i + \underbrace{\dot{\hat{e}}_i}_{\omega \times \hat{e}_i} r'^i = \vec{v}' + \vec{\omega} \times \vec{r}'$

or $\left(\frac{d}{dt}\right)_{S_0} = \left(\frac{d}{dt}\right)_S + \vec{\omega}\times \ldots$ $\qquad$ note: $\vec{\omega} = \vec{\omega}'$ always.

$\vec{a} = \left(\frac{d}{dt}\right)_{S_0}(\vec{v}' + \vec{\omega}\times\vec{r}') = \left(\frac{d}{dt}\right)_S(\vec{v}' + \vec{\omega}\times\vec{r}') + \vec{\omega}\times(\vec{v}' + \vec{\omega}\times\vec{r}')$

$$= \vec{a}' + \dot{\vec{\omega}} \times \vec{r}' + (\vec{\omega} \times \dot{\vec{r}}' + \vec{\omega} \times \vec{v}') + \omega \times \omega \times \vec{r}'$$

* General transformation: $Oxyz$ to $O'x'y'z'$ w/ $\hat{e}_1 \hat{e}_2 \hat{e}_3$:

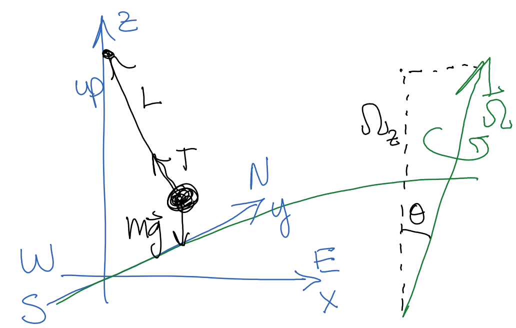
$$\vec{v} = \vec{v}' + \underbrace{\vec{\omega} \times \vec{r}'}_{\text{rotational}} + \underbrace{\vec{V}}_{\text{translation}}$$

$$\vec{a} = \vec{a}' + \underbrace{\vec{\alpha} \times \vec{r}'}_{\text{ang. accel.}} + \underbrace{2\vec{\omega} \times \vec{v}'}_{\text{Coriolus}} + \underbrace{\vec{\omega} \times \vec{\omega} \times \vec{r}'}_{\text{centripetal}} + \underbrace{\vec{A}}_{\text{translation}}$$

* Inertial forces:

$$m\vec{a}' = \vec{F} - \underbrace{m\alpha r \hat{\phi}}_{\text{tangential}} - \underbrace{2m\vec{\omega} \times \vec{v}'}_{\text{Coriolus}} + \underbrace{\frac{mv'^2}{\rho'}\hat{\rho}}_{\text{centrifugal}} - m\vec{A}$$

* Example: Focault Pendulum



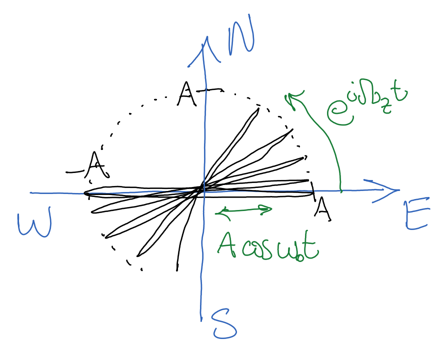
$$m\vec{a}' = \vec{T} + \underbrace{\underbrace{m\vec{g}_0 - m\vec{\Omega} \times \vec{\Omega} \times \vec{r}}_{m\vec{g}}}_{} - 2m\vec{\Omega} \times \vec{v}$$

$T_z \cong T = mg \qquad T_x = -\frac{mgx}{L} \qquad T_y = -\frac{mgy}{L}$

let $\xi = x + iy \qquad \ddot{\xi} = -\omega_0^2 \xi - 2i\Omega_z \dot{\xi} \qquad \omega_0^2 = g/L \qquad \Omega_z = \Omega\cos\theta$

$= e^{i\alpha t} \qquad \alpha^2 - 2i\Omega_z \alpha + \omega_0^2 = 0$

$$\alpha = \Omega_z \pm \sqrt{\cancel{\Omega_z^2} + \omega_0^2} = \Omega_z \pm \omega_0$$



$$\xi = e^{i(\Omega_z \pm \omega_0)t} \rightarrow e^{i\Omega_z t}(C_1 e^{i\omega_0 t} + C_2 e^{-i\omega_0 t})$$

$$= \underbrace{e^{i\Omega t}}_{\text{precession}}(\underbrace{\tilde{A}\cos\omega t}_{\text{oscillation}} + \cancel{\tilde{B}\sin\omega t})$$

$\tilde{A} = C_1 + C_2 \qquad \tilde{B} = i(C_1 - C_2)$

$= Ae^{i\phi_0} \qquad$ let $\phi_0 = 0$