L42 Summary

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0. Mathematics
    - ODEs (Matlab)
        - Leapfrog, Eigenfunctions ($\partial_x\, e^{ikx} = ik\, e^{ikx}$)
    - Linear spaces
        - Matrix multiplication, inner/outer products
        - Index notation, dot (δ_{ij}), cross (ϵ_{ijk}) products
        - Operators: Projections, Rotations, Stretches
        - Reciprocal basis, adjoint vectors
        - Continuous linear function spaces
    - Generalized coordinates, differential geometry
        - Coordinates, Co[ntra]variant basis ($\vec{b}_i, \tilde{b}^j$) / components
        - Metric (g_{ij}), Connection ($\vec{\Gamma}_{ij}$) -> Curvature ($R^i{}_{jkl}$)
    - Vector calculus and PDEs
        - Nabla (∇): gradient, curl, divergence -> Laplacian
        - FTVC (potentials), Helmholtz (L/T separations)
    - Calculus of variations
        - Functional derivative ($\frac{\delta}{\delta x} = -\frac{d}{dt}\frac{\partial}{\partial \dot{x}} + \frac{\partial}{\partial x}$)
1. General Mechanics
    - Kinematics: Degrees of Freedom (DoF)
        - Time (t), Position/Velocity/Acceleration ($\vec{r},\ \vec{v} = \dot{\vec{r}},\ \vec{a} = \dot{\vec{v}}$)
        - Canonical momentum ($p = \partial L/\partial \dot{q}$), phase space $a = (x, p)$
        - Rotation ($\overrightarrow{d\theta}, \vec{\omega}, \vec{\alpha}$), Euler angles ($\psi, \theta, \phi$)
    - Dynamics: Equation of Motion (EoM)
        - Newtons laws: (inertia, $F = ma$)
        - Lagrangian mechanics ($\delta L/\delta x = 0$): NII in covariant components

- Hamilton's principle ($\delta L/\delta x = 0$) Euler-Lagrange equations
- Hamilton's eq's ($\dot{x} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial x}$) symplectic symmetry ($\dot{a} = M\nabla_a H$)
- Rotational vs linear motion: Euler's equations
- Noninertial frames: Coriolis, Centrifugal forces

- Continuum mechanics
  - DoF: wave function (ω, k), superposition $A(k)$
  - EoM: wave equation, dispersion relation $v_\phi = \omega(k)/k = \sqrt{T/\mu}$
  - BC: external, internal (continuity), impedance $F = Zv$ $Z = \sqrt{T\mu}$
- Conserved currents
  - Cyclic coordinates ($\dot{p} = 0$), first integrals of motion
  - Noether's theorem (symmetry, current)
  1. Momentum ($\vec{p} = m\vec{v}$), 1st moment of mass
  2. Angular momentum ($\vec{l} = \vec{r} \times \vec{p}$), cannonical conjugate
  3. Energy: kinetic ($T = \frac{1}{2}mv^2$), potential (V) conservative forces
- Multiparticle systems
  - Moments of mass: (M, R_{cm}, I), CM particle (separation of p, L, T)
  - Reduced mass particle ($\mu^{-1} = m_1^{-1} + m_2^{-1}$)

2. Applications

0. Ballistic motion

   a. Constant acceleration, Damped motion

1. Oscillations

   a. Magnetic field, Hooke's law, Precession

   b. Impedance analogy (Z): electrical tank circuit

   c. Systems of particles: eigenfrequencies, normal modes

   d. Waves: string, waveguides, gravity, elastic, -> E&M, QM, GR

2. Central Forces

   a. Inverse-square law: Kepler orbits

    b. Cross sections
3. Rotational Motion
    a. Free precession
    b. Spinning top