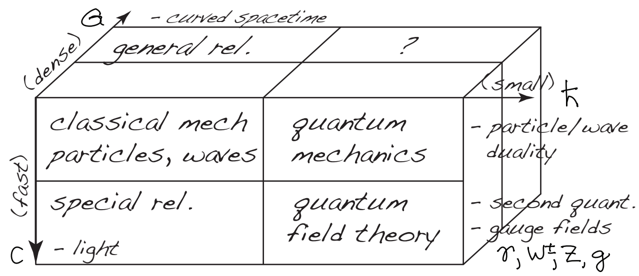
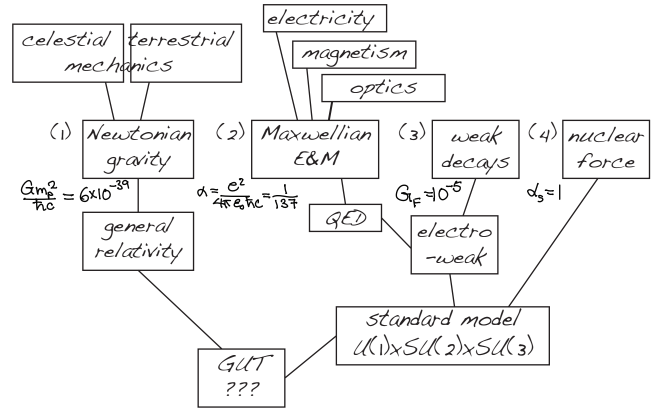
Survey of Electromagnetism

* Realms of Mechanics



~ E&M was second step in unification
~ the stimulus for special relativity
~ the foundation of QED -> standard model

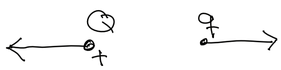
* Unification of Forces



* Electric charge (duFay, Franklin)

~ +,- equal & opposite (QCD: r+g+b=0)
~ $e = 1.6\times10^{-19}$ C, quantized ($q_n < 2\times10^{-21}$ e)
~ locally conserved (continuity)

* Electric Force (Coulomb, Cavendish)



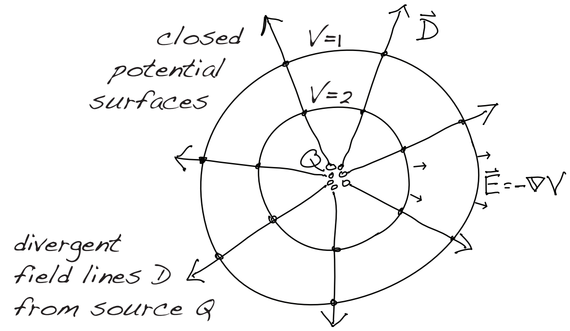
$$\vec{F} = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}}_{\vec{E}} \cdot q$$

* Electric Field (Faraday)

~ action at a distance vs. locality
field "mediates" or carries force
extends to quantum field theories
~ field is everywhere always $\vec{E}(x, t)$
differentiable, integrable
field lines, equipotentials
~ powerful techniques
for solving complex problems

* Electric potential

$\underbrace{\vec{F}}_{\text{force}} = q\underbrace{\vec{E}}_{\text{field}}$	$\vec{F} = m\underbrace{\vec{g}}_{\text{grav. field}}$
$\underbrace{U}_{\text{energy}} = q\underbrace{Ed}_{\text{potential}}$	$U = m\underbrace{gh}_{\text{"danger"}}$



* Field lines / Flux

~ E is tangent to the field lines
Flux = # of field lines
~ density of the lines = field strength
D is called "electric flux density"
~ note: $\frac{A}{r^2} = \Omega$ independent of distance

$$\boxed{\Phi_D = \int D \cdot d\vec{a}}$$

electric flux
flows from
(+) ⟶ (−)

$$\vec{D} = \varepsilon\vec{E} = \Phi_D/A$$

all flux lines begin at +
and end at - charge

* Equipotential surfaces / Flow

~ no work done to field lines
Equipotentials = surfaces of const energy
~ work is done along field line
Flow = # of potential surfaces crossed

$$\boxed{\mathcal{E}_E \equiv \int \vec{E} \cdot d\vec{\ell}}$$

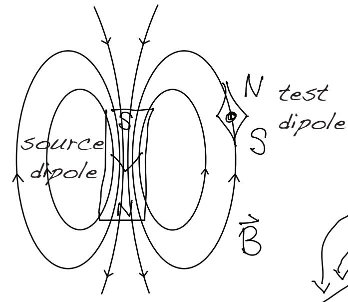
$$V = -\mathcal{E}_E$$

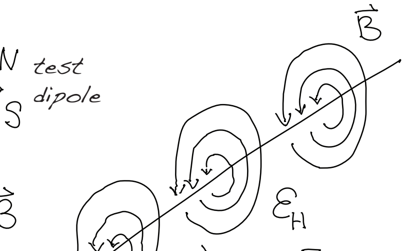
$$E = -\nabla V$$

~ potential if flow
is independent of path
~ circulation or EMF in a closed loop

* Magnetic field
- ~ no magnetic charge (monopole)
- ~ field lines must form loops
- ~ permanent magnetic dipoles first discovered

torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$
energy: $U = -\vec{\mu} \cdot \vec{B}$
force: $\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$



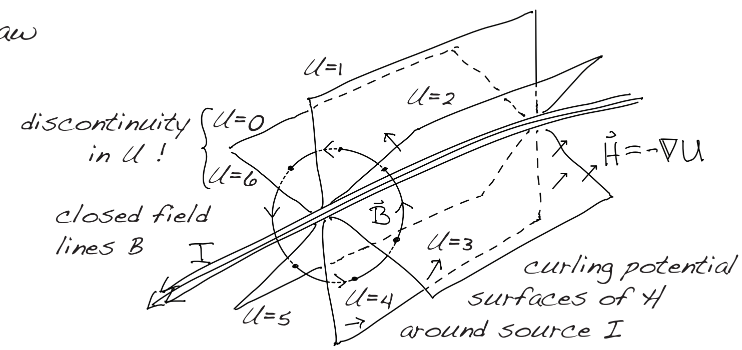


$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

- ~ electric current shown to generate fields (Oersted, Ampere)
- ~ magnetic dipoles are current loops
- ~ Biot-Savart law - analog of Coulomb law

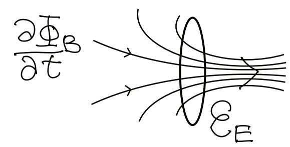
$$\vec{F} = \int I' d\vec{l} \times \underbrace{\frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}}_{\vec{B}}$$

- ~ B = flux density
- ~ H = field intensity $\quad \vec{B} = \mu \vec{H} = \Phi_B / A$



* Faraday law
- ~ opposite of Orsted's discovery: changing magnetic flux induces potential (EMF)
- ~ electric generators, transformers

$$\mathcal{E}_E = -\frac{\partial \Phi_B}{\partial t}$$



* Maxwell equations
- ~ added displacement current - D lines have +/- charge at each end
- ~ changing diplacement current equivalent to moving charge
- ~ derived conservation of charge and restored symmetry in equations
- ~ predicted electromagnetic radiation at the speed of light $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$$+ \xrightarrow{\Phi_D} -$$

$$I_d = \frac{\partial \Phi_D}{\partial t}$$

Maxwell equations

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \times \vec{E} + \partial_t \vec{B} = \vec{0}$$
$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{H} - \partial_t \vec{D} = \vec{J}$$

$$\boxed{\begin{array}{ll} \Phi_D = Q_{encl} & \Phi_B = 0 \\ \mathcal{E}_E = -\frac{\partial \Phi_B}{\partial t} & \mathcal{E}_H = I_{encl} + \frac{\partial \Phi_D}{\partial t} \end{array}}$$

Constitutive equations

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{B} = \mu \vec{H} \qquad \vec{J} = \sigma \vec{E}$$

Lorentz force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = \int (\rho \vec{E} + \vec{J} \times \vec{B})$$

Continuity

$$\nabla \cdot \vec{J} + \partial_t \rho = 0$$

Potentials

$$\vec{E} = -\nabla V - \partial_t \vec{A} \qquad \vec{B} = \nabla \times \vec{A}$$

Gauge transformation

$$V \to V - \partial_t \lambda \qquad \vec{A} \to \vec{A} + \nabla \lambda$$

$$\begin{tikzcd}
 & (0) & (1) & (2) & (3) & (4) \\
0 \arrow[r] & \lambda \arrow[r, "d"] & (V, \vec{A}) \arrow[r, "d"] & (\vec{E}, \vec{B}) \arrow[r, "d"] \arrow[d, shift right, "\varepsilon"'] \arrow[dr, "\sigma"] & 0 & \\
 & & (\ , \mu) \arrow[r, dash, "d"] & (\vec{D}, \vec{H}) \arrow[r, "d"] \arrow[u, shift right, "\mu"'] & (\rho, \vec{J}) \arrow[r, "d"] & 0
\end{tikzcd}$$

Wave equation $\quad -\square^2 (V, \vec{A}) = (\rho/\varepsilon, \mu \vec{J})$