

Section 2.5 - Conductors

* conductor

~ has abundant "free charge", which can move anywhere in the conductor

* types of conductors

i) metal: conduction band electrons, ~ 1 / atom

ii) electrolyte: positive & negative ions

* electrical properties of conductors

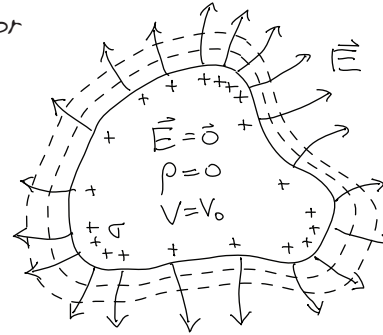
i) electric field = 0 inside conductor

therefore $V = \text{constant}$ inside conductor

ii) electric charge distributes itself

all on the boundary of the conductor

iii) electric field is perpendicular to the surface just outside the conductor



	inside	outside
ρ	0	σ
$\vec{E}$	$\vec{0}$	$\frac{\sigma \hat{n}}{\epsilon_0}$
V	V_0	$V_0 + \delta$

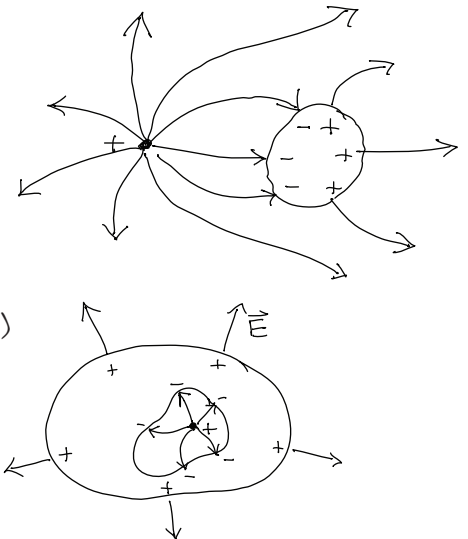
* induced charges

~ free charge will shift around charge on a conductor

~ induces opposite charge on near side of conductor to cancel out field lines inside the conductor

~ Faraday cage: external field lines are shielded inside a hollow conductor

~ field lines from charge inside a hollow conductor are "communicated" outside the conductor by induction (as if the charge were distributed on a solid conductor)
compare: displacement currents, sec. 7.3



* electrostatic pressure

~ on the surface: $\vec{F}_A \equiv \vec{f} = \sigma (\vec{E}_{\text{patch}} + \vec{E}_{\text{other}}) = \frac{1}{2} \sigma (\vec{E}_{\text{inside}} + \vec{E}_{\text{outside}})$

~ for a conductor: $\vec{E}_{\text{inside}} = 0$ $\vec{E}_{\text{out}} = \frac{\sigma}{\epsilon_0}$ $P = f = \frac{\sigma^2}{2\epsilon_0} = \frac{\epsilon_0}{2} E^2$

~ note: electrostatic pressure corresponds to energy density $P \approx w$
both are part of the stress-energy tensor

Capacitance

* capacitance

- ~ a capacitor is a pair of conductors held at different potentials, stores charge
- ~ electric FLOW from one conductor to the other equals the POTENTIAL difference
- ~ electric FLUX from one conductor to the other is proportional to the CHARGE

$$C = Q/\Delta V = \frac{\epsilon_0 \Phi_E}{\mathcal{E}_E} \quad Q = \int da \sigma = \int d\vec{a} \cdot \epsilon_0 \vec{E} = \epsilon_0 \Phi_E \quad (\text{closed surface})$$

$$\Delta V = \int d\vec{l} \cdot \vec{E} = \mathcal{E}_E \quad (\text{open path})$$

- ~ this pattern repeats itself for many other components: resistors, inductors, reluctance (next semester)

* work formulation

$$W = \frac{1}{2} QV = \frac{1}{2} CV^2 = \int \frac{\epsilon_0}{2} E^2 d\tau$$

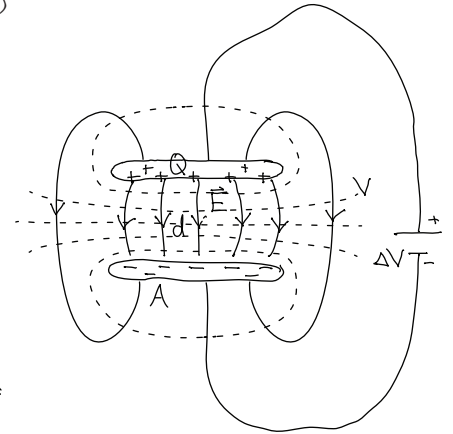
$$= \frac{\epsilon_0}{2} \text{flux} \cdot \text{flow}$$

$$C = \frac{2W}{V^2} = \frac{\epsilon_0}{V^2} \int E^2 d\tau = \frac{\epsilon_0}{2} \frac{\text{flux} \cdot \text{flow}}{\text{flow} \cdot \text{flow}}$$

* ex: parallel plates

$$C = \frac{\epsilon_0 \Phi_E}{\mathcal{E}_E}$$

$$= \frac{\epsilon_0 EA}{Ed} = \frac{\epsilon_0 A}{d}$$



* capacitance matrix

- ~ in a system of conductors, each is at a constant potential
- ~ the potential of each conductor is proportional to the individual charge on each of the conductors
- ~ proportionality expressed as a matrix coefficients of potential P_{ij} or capacitance matrix C_{ij}

$$V_i = P_{ij} Q_j \quad \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$$

$$Q_i = C_{ij} V_j$$

$$-\nabla^2 V = \rho/\epsilon_0 \quad V(\vec{r}) \propto Q$$

