

Section 4.3 - Electric Displacement $\vec{D}$

- * reviews: parallels between E and P
 - ~ what are the units of $\epsilon_0 \vec{E}$? $\vec{P}$?
 - ~ both are vector fields (functions of position)
 - ~ the field lines (flux) are associated with charge (Dr. Jekyll or Mr. Hyde??)
 - ~ the two fields are related: E induces P in a dielectric

$$\Phi_{\epsilon_0 E} = Q \quad \nabla \cdot \epsilon_0 \vec{E} = \rho \quad \hat{n} \cdot \Delta \epsilon_0 \vec{E} = \sigma \quad \text{total charge}$$

$$\textcircled{+} \Phi_P = -Q_b \quad \textcircled{+} \nabla \cdot \vec{P} = -\rho_b \quad \textcircled{+} \hat{n} \cdot \Delta \vec{P} = -\sigma_b \quad \textcircled{-} \text{bound charge}$$

$$\Phi_D = Q_f \quad \nabla \cdot \vec{D} = \rho_f \quad \hat{n} \cdot \Delta \vec{D} = \sigma_f \quad \text{free charge}$$

$$D_2^\perp - D_1^\perp = \sigma_f$$

- * new field: $\vec{D}$ = "electric displacement"

~ defined by the "constitutive equation": $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

~ associated with the free charge:

lines of $\vec{D}$ flux go from (+) to (-) free charge

~ iterative cycle:

a) free charge generates E

b) E causes P , displaced bound charge $\rho_b \sigma_b$

c) the field from bound charge modifies E

~ direct calculation procedure with $\vec{D}$

a) calculate $\vec{D}$ directly from free charge only

b) obtain P from $\vec{D}$ using constitutive relation

c) the electric field is: $\epsilon_0 \vec{E} = \vec{D} - \vec{P}$

- * differences between $\epsilon_0 E$, P , and D :

~ equipotentials associated with force $\vec{F} = q\vec{E}$ only for the electric field

~ ρ generates E , but P induces ρ_b

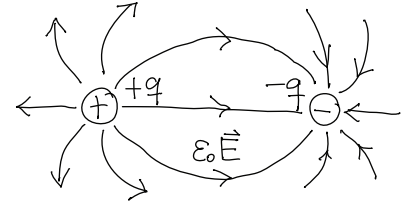
$$\sim \epsilon_f = 0 \quad \nabla \times \vec{E} = \vec{0} \quad \hat{n} \times \Delta \vec{E} = \vec{0}$$

$$\sim \vec{E} = \int \frac{dq \hat{r}}{4\pi \epsilon_0 r^2} \quad \vec{E}_b = \int \frac{dq_b \hat{r}}{4\pi \epsilon_0 r^2} \quad \vec{E}_f = \int \frac{dq_f \hat{r}}{4\pi \epsilon_0 r^2}$$

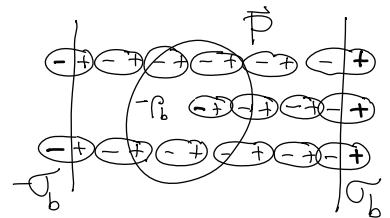
note: not P or D in these formulas!

- * you need both $\nabla \cdot \vec{D} = \rho_f$ and $\nabla \times \vec{E} = \vec{0}$ to solve!

Electric field " E "

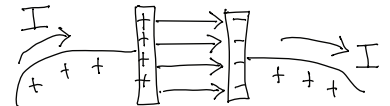


Polarization " P "

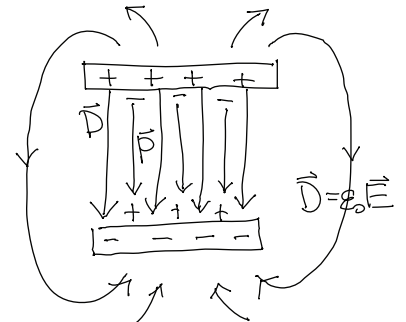


"Displacement current" (Maxwell)

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$



$$I_d = \int \vec{J}_d \cdot d\vec{a} = \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{a} = \frac{\partial \Phi_D}{\partial t}$$



$$\epsilon_0 E = D - P \quad \text{inside}$$

$$\epsilon_0 E = D \quad \text{outside}$$