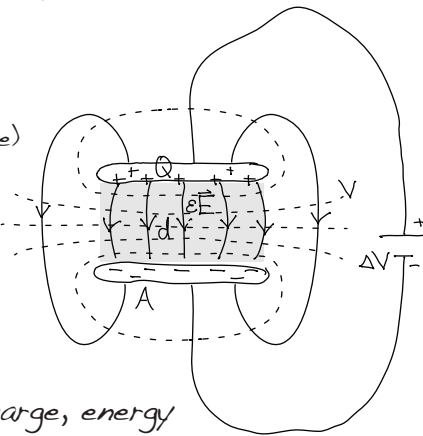


Section 4.4.3 - Energy in Dielectric Systems

* capacitance = flux/flow

$$C = \frac{Q}{\Delta V} = \frac{\Phi_D}{E_F} = \frac{\epsilon \Phi_E}{E_E} \approx \frac{\epsilon A}{d} \quad Q = \int d\vec{a} \cdot \vec{D} = \Phi_D \text{ (closed surface)}$$

$$\epsilon_0 \vec{E} \rightarrow \vec{D} \quad \epsilon_0 \rightarrow \epsilon = \epsilon_0 \epsilon_r \quad \Delta V = \int d\vec{l} \cdot \vec{E} = E_E \text{ (open path)}$$



* energy in a capacitor

$$W = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

~ where does the 1/2 come from?

~ ϵ_r (dielectric const) enhancement factor of capacitance, charge, energy

* energy in the electric field = flux x flow $W = \frac{\epsilon_0}{2} \int E^2 d\tau \rightarrow \frac{\epsilon}{2} \int E^2 d\tau = \frac{1}{2} \int \vec{D} \cdot \vec{E} d\tau$

~ proof: $V \rightarrow \vec{E}$ or $\vec{D} \rightarrow \rho_f$

$$\Delta W = \int \Delta \rho_f V d\tau = \int (\nabla \cdot \Delta \vec{D}) d\tau = \oint d\vec{a} \cdot (\Delta \vec{D} V) - \int \Delta \vec{D} \cdot \nabla V d\tau = \int \Delta \vec{D} \cdot \vec{E} d\tau$$

~ for a linear dielectric (linear materials),

$$\Delta W = \int \epsilon \Delta \vec{E} \cdot \vec{E} d\tau = \int \frac{\epsilon}{2} \Delta E^2 d\tau = \Delta \frac{1}{2} \int \vec{D} \cdot \vec{E} d\tau$$

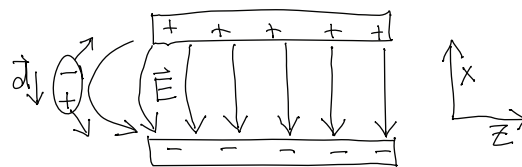
$$W = \frac{1}{2} \int \vec{D} \cdot \vec{E} d\tau$$

* forces on dielectrics

~ force of a fringe field on a dipole

~ due to $\frac{\partial E_z}{\partial x}$ which equals $\frac{\partial E_x}{\partial z}$

$$\vec{F} = -\nabla W = \nabla(\vec{d} \cdot \vec{E}) = \vec{d} \times (\nabla \times \vec{E}) + (\vec{d} \cdot \nabla) \vec{E}$$

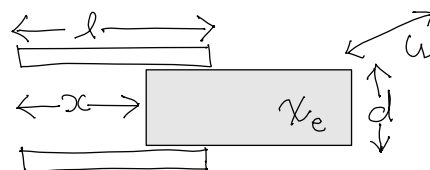


* Example: dielectric being pulled into a capacitor

$$C = C_1 + C_2 = \epsilon_0 \frac{xw}{d} + \epsilon_0 (1 + \chi_e) \frac{(l-x)w}{d} = \frac{\epsilon_0 w}{d} (\epsilon_r l - \chi_e x)$$

$$F = -\nabla W = -\frac{d}{dx} \frac{1}{2} CV^2 = -\frac{d}{dx} \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{C^2} \frac{dC}{dx}$$

$$= \frac{1}{2} V^2 \frac{dC}{dx} = -\frac{\epsilon_0 w \chi_e}{2d} V^2$$



~ with constant V , the force would be the same $F = -\nabla W$
but in this case W would increase