Dipole in dielectric & External field & Free charge $\sigma = \sigma_0 \cos\theta$

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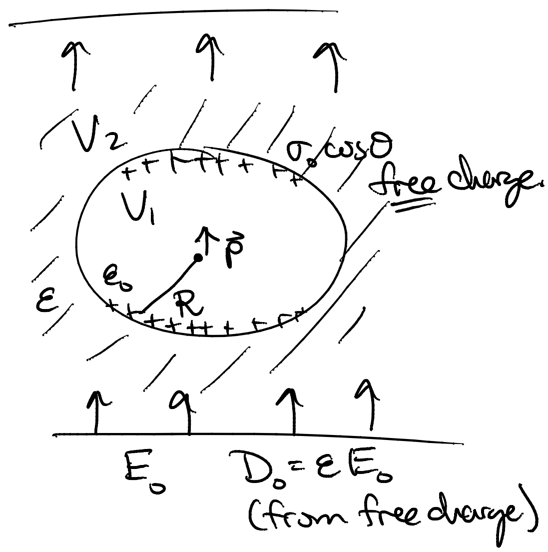
9:17 AM



$$V = \sum_{l=0}^{\infty} (a_l r^l + b_l r^{-l-1}) P_l(\cos\theta)$$

$$V_{\vec{p}} = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} = \underbrace{\frac{p}{4\pi\epsilon_0}}_{b_1}\underbrace{r^{-2}\cos\theta}_{r^{-1-1}P_1} \qquad l=1$$

$Q^{(1)}_{int}$

$$V_{E_0} = -\int E_0\,dz = -E_0 z = \underbrace{-E_0}_{a_1}\underbrace{r^1\cos\theta}_{r^1 P_1} \qquad l=1$$

$Q^1_{\cancel{ext}} 4\pi\epsilon_0$

$$V_1 = \frac{p}{4\pi\epsilon_0} r^{-2} P_1 + \sum_{l=0}^{\infty} a_l r^l P_l$$

$$V_2 = -E_0 r P_1 + \sum_{l=0}^{\infty} b_l r^{-l-1} P_l$$

* Apply boundary conditions:

$$\Delta V|_\partial = 0: \quad \left(-E_0 R P_1 + \sum_{l=0}^{\infty} b_l R^{-l-1} P_l\right) - \left(\frac{p}{4\pi\epsilon_0} R^{-2} P_1 + \sum_{l=0}^{\infty} a_l R^l P_l\right) = 0$$

$$-\Delta \varepsilon \frac{\partial V}{\partial n}\Big|_\partial = 0: \quad -\varepsilon_r\left(-E_0 P_1 + \sum_{l=0}^{\infty}(-l-1) b_l R^{-l-2} P_l\right) + \left(\frac{p}{4\pi\epsilon_0}(-2) R^{-3} P_1 + \sum_{l=0}^{\infty} l a_l R^{l-1} P_l\right) = \sigma_0/\epsilon_0\, P_1$$

* Separate out components:

if $l \neq 1$: $\quad b_l R^l - a_l R^{-l-1} = 0$

$\quad -\varepsilon_r(-l-1) b_l R^{-l-2} + l a_l R^{l-1} = 0 \quad \Rightarrow a_l = b_l = 0 \quad$ no "source" term

$l=1$: $\quad -E_0 \cancel{R} + b_1 R^{-2\,3} - \frac{p}{4\pi\epsilon_0} R^{-2\,3} - a_1 \cancel{R} = 0$

$$\varepsilon_r E_0 + 2\varepsilon_r \underbrace{b_1 R^{-3}}_{b'} - 2\underbrace{\frac{p}{4\pi\epsilon_0} R^{-3}}_{p'} + a_1 = \underbrace{\sigma_0/\epsilon_0}_{\sigma'}$$

$$b' - a_1 = E_0 + p' \qquad (1)$$

$$2\varepsilon_r b' + a_1 = -\varepsilon_r E_0 + 2p' + \sigma' \qquad (2)$$

source terms on right

$(2) - 2\varepsilon_r(1)$: $\quad (1+2\varepsilon_r)\, a_1 = (-\varepsilon_r - 2\varepsilon_r) E_0 + (2-2\varepsilon_r) p' + \sigma'$

$(2) + (1)$: $\quad (1+2\varepsilon_r)\, b_1 = (-\varepsilon_r + 1) E_0 + (2+1) p' + \sigma'$