University of Kentucky, Physics 416G
Problem Set #5, Rev. A, due Friday, 2014-10-17

1. (continuation of HW 4 #1). Consider the electric potential $V(\boldsymbol{r})$ defined by $\boldsymbol{E} = -\nabla V$.

a) Show using one of the boundary theorems that the above definition is equivalent to $V(\boldsymbol{r}) = -\mathcal{E}_E \equiv -\int_{\boldsymbol{r}_0}^{\boldsymbol{r}} \boldsymbol{E}\cdot \boldsymbol{dl}$. We call $\boldsymbol{r}_0$ the ground point and $V(\boldsymbol{r}_0) = 0$ the ground potential. In residential wiring, this point ties to a physical rod driven deep into the earth.

b) Show that (III) implies that $V(\boldsymbol{r})$ exists, and that (III) $\Rightarrow$ (V). Note that the two first-order differential equations (III) are equivalent to the single second-order differential equation in V.

c) Using the definition in a), integrate the flow of (I) over an arbitrary path in $\boldsymbol{r}$ starting at $r_0 = \infty$ to show that (I) $\Rightarrow$ (IV). Conversely, calculate the gradient of (IV) to derive equation (I) from equation (IV). We say that $\hat{\boldsymbol{\imath}} \cdot \boldsymbol{dl}/\imath^2 = -d(1/\imath)$ is a perfect (exact) differential.

d) Calculate the Laplacian in (IV), both directly and from the divergence of (I) (HW 4), to show that (IV) $\Rightarrow$ (V) and (I) $\Rightarrow$ (V).

e) Show that (IV) $\Leftarrow$ (V) by the decomposition of $\rho(\boldsymbol{r})$ into an integral ('forest') over $đq'$ of delta functions ('poles') and applying $\nabla^{-2}\delta^3(\hat{\boldsymbol{\imath}})$ to get a Green's function (point potential) for each charge.

f) Rework the proof of the Helmholtz theorem and use it to show that (III) $\Rightarrow$ (I). Note that you need to use both equations of (III) in the Helmholtz theorem to obtain (I). The calculation pieces together parts b), e), and c), involving the tortuous path (III) $\Rightarrow$ (V) $\Rightarrow$ (IV) $\Rightarrow$ (I). Use the Helmholtz theorem to prove one more time that $\boldsymbol{E}(\boldsymbol{r})$ admits a potential $V(\boldsymbol{r})$.

g) Identify the six formulas used to calculate any field in the derivative chain $V \xrightarrow{d} \boldsymbol{E} \xrightarrow{\delta} \rho$ in terms of either of the other two fields.

Bonus: repeat above for the new formulation (VI): show the four steps of (I) $\Leftrightarrow$ (VI) $\Leftrightarrow$ (V).

2. Sketch the field lines and potentials in the xy-plane for: **a)** a system of three point charges: charge $+q$ at $(0, 0)$, charge $-q$ at $(1, 0)$, and charge $-q$ at $(0, 1)$; **b)** charge $+q$ at $(a, 0, 0)$ and charge $-2q$ at $(-a, 0, 0)$. For b), how many points in space are there where $\boldsymbol{E} = \boldsymbol{0}$? Calcuate their positions, if any.

3. a) Show that the potential due to two parallel line charges of opposite polarity is $V = \frac{\lambda}{2\pi\epsilon_0} \ln(\frac{s_1}{s_2})$ where s_1 is the distance from the field point to the negative line, and s_2 is the distance to the positive line. Hint: use the midpoint of the two line charges as ground.

b) Determine the shape and equation of the equipotential surfaces, assuming that the negative line charge is along the z-axis at $(x, y) = (-a, 0)$ and the positive line charge is at $(+a, 0)$.

4. The Spallation Neutron Source (SNS) at Oak Ridge National Laboratory accelerates protons to an energy of 1 GeV, to spall (smash) neutrons out of a mercury target, yielding on average 30 free neutrons per incident proton. At the nominal beam current is 1.4 mA, how many neutrons are 'produced' per second? How much power is dumped into the target?

5. Calculate the potential energy of a cube of length a on each side with a point charge q at each of the eight vertices.

Also, Griffiths 3ed[4ed] Ch. 2, #20[20], 21[21], 26[26], 32[34], 34[36], 39[43], 46[50], bonus: 48[53], 49[54], 51[56], 52[57].