University of Kentucky, Physics 416G
Problem Set #10, Rev. A, due Monday, 2014-12-08

1. An infinite **cylindrical rod** of radius R centered in the xy-plane and extending along the z-axis has a uniform polarization $\boldsymbol{P} = P\hat{\boldsymbol{x}}$.

a) Calculate the bound surface charge density σ_b.

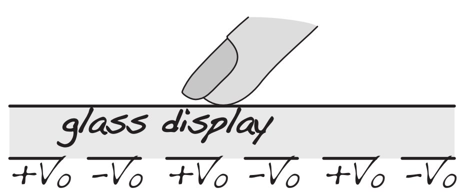
b) Calculate $\boldsymbol{E}_b$ (due to the bound charge) inside and outside the cylinder as a function of $\boldsymbol{P}$.

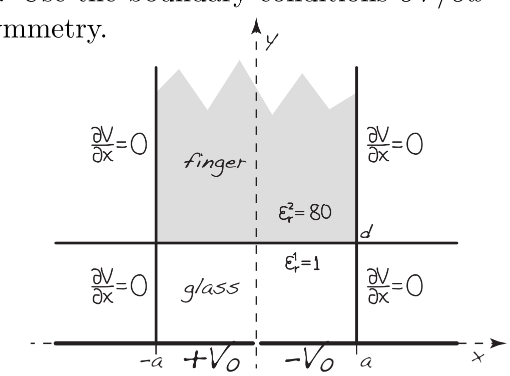
c) Suppose the rod is placed in an external electric field $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$, and that $\boldsymbol{P} = \chi_e\epsilon_0\boldsymbol{E}$, where $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_b$ is the total (external plus bound) electric field inside the rod. Calculate the values of $\boldsymbol{E}$, $\boldsymbol{P}$, and $\boldsymbol{D}$ inside and outside the rod.

d) Check the boundary conditions $\epsilon_0 E_{2n} - \epsilon_0 E_{1n} = P_n$ and $D_{2n} - D_{1n} = 0$ for above.

e) Solve the same problem (part c) as a boundary value problem with dielectric constant $\epsilon_r = 1 + \chi_e$ inside the rod and $\sigma_f = 0$ on the surface (only bound surface charge).

2. A **capacitive touch-screen**, has strips of the transparent conductive coating indium tin oxide (ITO) of width a under of a glass plate of thickness d. The strips are at alternating potentials $\pm V_0$. Your finger (a dielectric) is detected by the change of capacitance when it touches the glass. We will model a single cell in 2-d (symmetric along the length of the strip) with period boundary conditions from the middle of one strip to the middle of the next. Thus the domain is a semi-infinite rectangle extending upward from the two half-strips. Use the boundary conditions $\partial V/\partial x = 0$ on the sides at $x = \pm a$ to ensure the correct periodic symmetry.





a) Solve the boundary value problem for the potential without a finger ($\epsilon_r^1 = \epsilon_r^2 = 1$). Note that in this case there is no discontinuity at the glass and the problem can be solved with a single region extending from $0 < y < \infty$. Calculate the first two nonzero terms of the Fourier expansion.

b) Solve for the potential when a finger (assume pure water, $\epsilon_r^2 \approx 80$) touches the display and fills the entire region beyond the glass. Check that your answer reduces to part a) as $\epsilon_r^2 \to \epsilon_r^1$.

c) Plot the equipotentials and field lines in the region with and without your finger.

d) Calculate the capacitance/length with and without your finger, as a function of d/a. Don't forget fields on both the top and bottom of the strips. What glass thickness do you recommend?

e) If the voltage on each wire is $\pm V_0 = \pm 2V$, what difference in charge must the device be able to measure, for $d = a = 3$ mm in a 1 cm long strip?

Also, Griffiths 3ed[4ed] Ch. 4, #16[16], 20[20], 21[21], 24[24], 28[28], 33[36], bonus: 29[29], 30[30], 32[35], 34[37].