Instructions: The exam is closed book and timed (50 minutes). Show all steps of calculations. Be careful to pace yourself; you may want to set up all integrals before evaluation. [65 pts total]

[24 pts] 1a. Proof. Derive Coulomb's law from Maxwell's static electric field equations. (Alternate proof for partial credit (12 pts): Show that the two differential and integral forms of Maxwell's equations are equivalent.)

Force   *Coulomb's law*

$$\vec{F} = q\vec{E} \qquad \boxed{\vec{E} = \int \frac{\mathchar'26\mkern-9mu d q\,\hat{\imath}}{4\pi\epsilon_0\,\imath^2}}$$

Integral   *Maxwell eq's*   *Differential*

Gauss   *Gauss*

$$\Phi_E = Q/\epsilon_0 \quad \Longrightarrow \quad \boldsymbol{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\mathcal{E}_E = 0 \quad \Longrightarrow \quad \boldsymbol{\nabla}\times\vec{E} = \mathbf{0}$$

$$\mathcal{E}_E = -\Delta V \qquad \qquad \vec{E} = -\boldsymbol{\nabla}V$$

Energy   *Potential*   *Poisson*

$$W = qV \qquad V = \int \frac{\mathchar'26\mkern-9mu d q}{4\pi\epsilon_0\,\imath} \qquad \nabla^2 V = -\rho/\epsilon_0$$
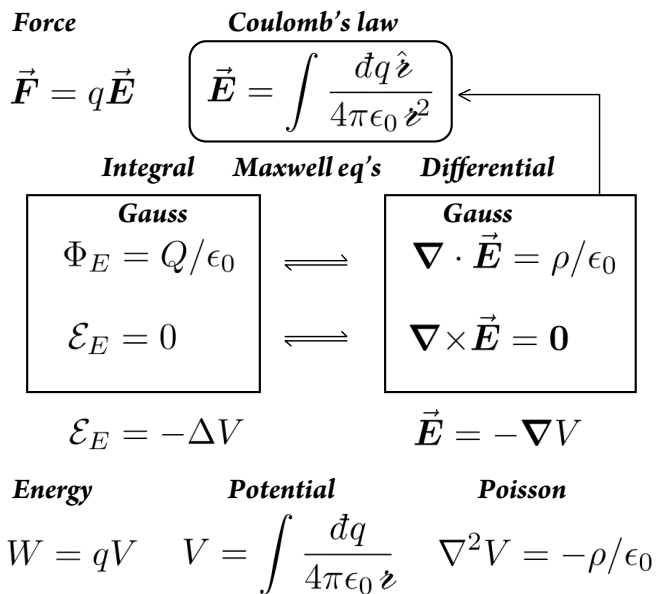
[6 pts] 1b. Fill in the 6 equations in the following triangle (V, $\vec{E}$, ρ) each in terms of each of the other two quantities).

$$V \quad \Longrightarrow \quad \vec{E}$$

$$\rho$$
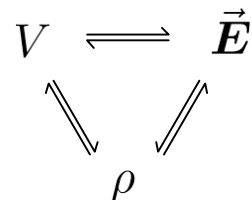
[15 pts] 2. Calculate the electric field at the origin from a distribution of total charge q spread evenly over the quarter-circle arc of radius R as shown in the diagram.
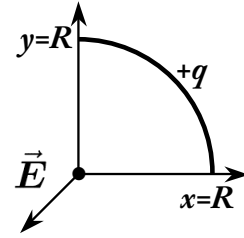
3. Two spheres of radius $R_2 > R_1$ carry charge $+q$ (outer) and $-q$ (inner).
[7 pts] a) Calculate the electric field everywhere in space.

[3 pts] b) Show that $E_{2n} - E_{1n} = \sigma/\epsilon_0$ across the outer sphere.

[7 pts] c) Calculate the electric potential everywhere in space.

[3 pts] d) Calculate the capacitance of the system.