University of Kentucky, Physics 416G
Formula Sheet

The following formulas will be provided if necessary on the exam.

Solutions of Laplace equation $\nabla^2 V = 0$:

- Separation of variables in cartesian coordinates:
  $V(x, y, z) = \exp(k_x x)\exp(k_y y)\exp(k_z z)$ where $k_x^2 + k_y^2 + k_z^2 = 0$,
  i.e. $V(x, y) = (A_n \exp(k_n x) + B_n \exp(-k_n x))(C_n \cos(k_n y) + D_n \sin(k_n y))$
  or $V(x, y) = (A_n \sin(k_n x) + B_n \cos(-k_n x))(C_n \exp(k_n y) + D_n \exp(-k_n y))$
- Separation of variables in cylindrical coordinates with z-symmetry and appropriate azimuthal boundary conditions:
  $V(s, \phi) = A_0 + B_0 \ln(s) + \sum_{m=0}^{\infty}(A_m s^m + B_m s^{-m})(C_m \cos(m\phi) + D_m \sin(m\phi))$.
- Separation of variables in spherical coordinates with *phi*-symmetry and appropriate polar boundary conditions:
  $V(r, \theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + B_\ell r^{-\ell-1})P_\ell(\cos\theta)$
  where $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$, etc.

Continuity boundary conditions: $V_1 = V_2$ and $\frac{\partial V_1}{\partial n} - \frac{\partial V_2}{\partial n} = \frac{\sigma}{\epsilon_0}$ evaluated on the boundary.

Multipole moments for charge distribution:

- General: $Q_\ell = \int đq' r'^\ell P_\ell(\cos\theta)$.
- Monopole: $q = \int đq'$.
- Dipole: $\vec{\boldsymbol{p}} = \int đq' \vec{\boldsymbol{r}}'$.
- Quadrupole: $Q_{ij} = \int đq'(3r_i' r_j' - \delta_{ij} r'^2)$
  i.e. $Q_{xx} = \int đq'(3x'x' - r'^2)$
  $Q_{xy} = \int đq'(3x'y')$, etc.