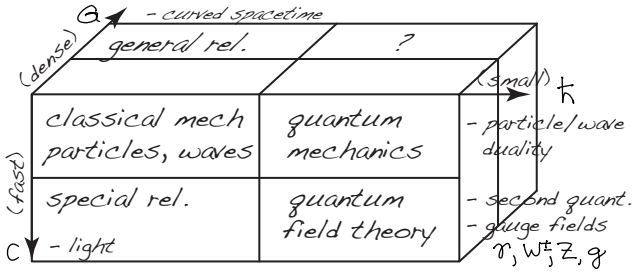
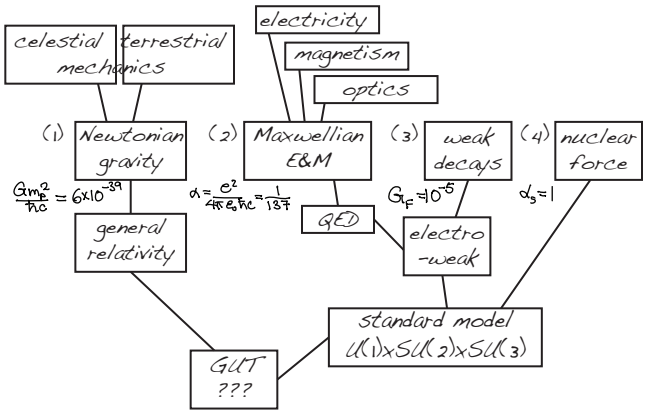
Survey of Electromagnetism

* Realms of Mechanics



~ E&M was second step in unification
~ the stimulus for special relativity
~ the foundation of QED -> standard model

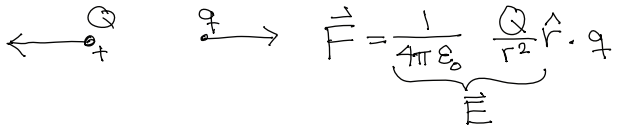
* Unification of Forces



* Electric charge (duFay, Franklin)

~ +,- equal & opposite (QCD: r+g+b=0)
~ $e = 1.6\times10^{-19}$ C, quantized ($q_n < 2\times10^{-21}$ e)
~ locally conserved (continuity)

* Electric Force (Coulomb, Cavendish)

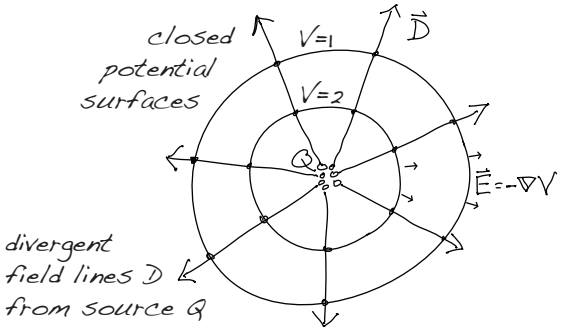


* Electric Field (Faraday)

~ action at a distance vs. locality
  field "mediates" or carries force
  extends to quantum field theories
~ field is everywhere always $\vec{E}(x,t)$
  differentiable, integrable
  field lines, equipotentials
~ powerful techniques
  for solving complex problems

* Electric potential

$\vec{F} = q\vec{E}$ (force, field)	$\vec{F} = m\vec{g}$ (grav. field)
$U = qEd$ (energy, potential)	$U = mgh$ ("danger")



* Field lines / Flux

~ E is tangent to the field lines
  Flux = # of field lines
~ density of the lines = field strength
  D is called "electric flux density"
~ note: $\frac{A}{r^2} = \Omega$ independent of distance

$$\Phi_D \equiv \int \vec{D}\cdot d\vec{a}$$

electric flux
flows from
(+) ⟶ (-)
all flux lines begin at +
and end at - charge

$$\vec{D} = \varepsilon\vec{E} = \Phi_D/A$$

* Equipotential surfaces / Flow

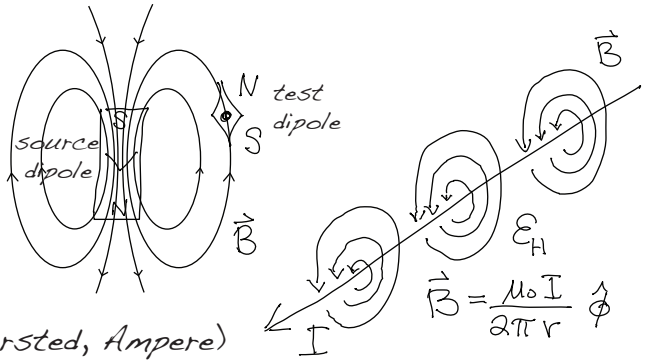
~ no work done to field lines
  Equipotentials = surfaces of const energy
~ work is done along field line
  Flow = # of potential surfaces crossed

$$\mathcal{E}_E \equiv \int \vec{E}\cdot d\vec{\ell} \qquad V = -\mathcal{E}_E$$

$$E = -\nabla V$$

~ potential if flow is independent of path
~ circulation or EMF in a closed loop

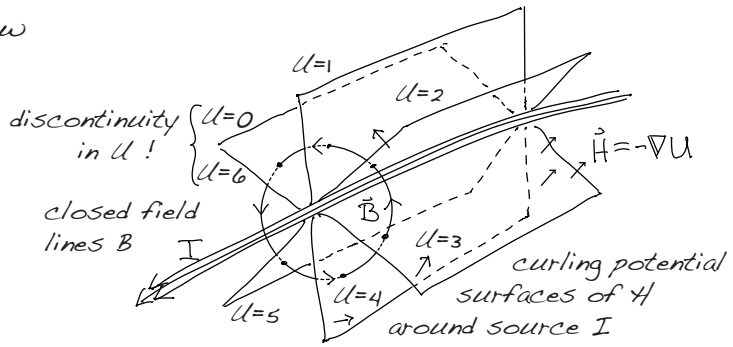
* Magnetic field
  - ~ no magnetic charge (monopole)
  - ~ field lines must form loops
  - ~ permanent magnetic dipoles first discovered
    torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$
    energy: $U = -\vec{\mu} \cdot \vec{B}$
    force: $\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$



  - ~ electric current shown to generate fields (Oersted, Ampere)
  - ~ magnetic dipoles are current loops
  - ~ Biot-Savart law - analog of Coulomb law

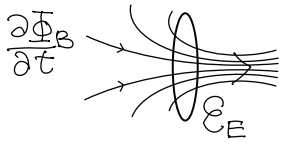
$$\vec{F} = \int I' d\vec{\ell} \times \underbrace{\frac{\mu_0}{4\pi}\int \frac{I d\vec{l} \times \hat{r}}{r^2}}_{\vec{B}}$$

  - ~ B = flux density
  - ~ H = field intensity $\vec{B} = \mu\vec{H} = \Phi_B/A$

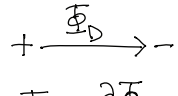


* Faraday law
  - ~ opposite of Orsted's discovery:
    changing magnetic flux induces potential (EMF)
  - ~ electric generators, transformers

$$\mathcal{E}_E = -\frac{\partial \Phi_B}{\partial t}$$



* Maxwell equations
  - ~ added displacement current - D lines have +/- charge at each end
  - ~ changing diplacement current equivalent to moving charge
  - ~ derived conservation of charge and restored symmetry in equations
  - ~ predicted electromagnetic radiation at the speed of light $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$



$$I_d = \frac{\partial \Phi_D}{\partial t}$$

Maxwell equations

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \times \vec{E} + \partial_t \vec{B} = \vec{0}$$
$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{H} - \partial_t \vec{D} = \vec{J}$$

Constitutive equations

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{B} = \mu \vec{H} \qquad \vec{J} = \sigma \vec{E}$$

Lorentz force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = \int (\rho \vec{E} + \vec{J} \times \vec{B})$$

Continuity

$$\nabla \cdot \vec{J} + \partial_t \rho = 0$$

Potentials

$$\vec{E} = -\nabla V - \partial_t \vec{A} \qquad \vec{B} = \nabla \times \vec{A}$$

Gauge transformation

$$V \to V - \partial_t \lambda \qquad \vec{A} \to \vec{A} + \nabla \lambda$$

$$\Phi_D = Q_{encl} \qquad \Phi_B = 0$$
$$\mathcal{E}_E = -\frac{\partial \Phi_B}{\partial t} \qquad \mathcal{E}_H = I_{encl} + \frac{\partial \Phi_D}{\partial t}$$

$$0 \xrightarrow{(0)} \lambda \xrightarrow{d} (V, \vec{A}) \xrightarrow{d} (\vec{E}, \vec{B}) \xrightarrow{d} 0 \xrightarrow{(3)} \qquad (4)$$

$$\varepsilon \downarrow \quad \uparrow \mu \quad \searrow \sigma$$

$$(\ , u) \xrightarrow{d} (\vec{D}, \vec{H}) \xrightarrow{d} (\rho, \vec{J}) \xrightarrow{d} 0$$

Wave equation $\quad -\Box^2 (V, \vec{A}) = (\rho/\varepsilon, \mu \vec{J})$