Section 2.1 - Coulomb's Law

* Electric charge (duFay, Franklin)
- ~ +,- equal & opposite (QCD: $r+g+b=0$)
- ~ $e=1.6\times10^{-19}$ C, quantized ($q_n < 2\times10^{-21}\,e$)
- ~ locally conserved (continuity)

Seventhly, Chance has thrown in my Way another Principle, more universal and remarkable than the preceding one, and which casts a new Light on the Subject of Electricity. This Principle is, that there are two distinct Electricities, very different from one another; one of which I call *vitreous Electricity*, and the other *resinous Electricity*. The first is that of Glass, Rock-Crystal, Precious Stones, Hair of Animals, Wool, and many other Bodies: The second is that of Amber, Copal, Gum-Lack, Silk, Thread, Paper, and a vast Number of other Substances.

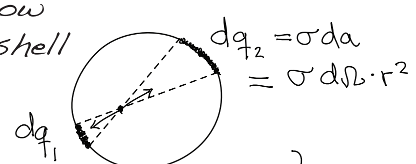
Charles François de Cisternay DuFay, 1734
http://www.sparkmuseum.com/BOOK_DUFAY.HTM

* only for static charge distributions (test charge may move but not sources)

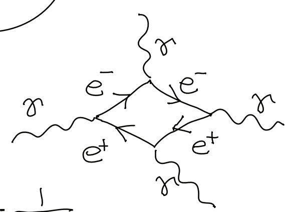
a) Coulomb's law $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}}$

b) Superposition $\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots$

- ~ Coulomb: torsion balance
- ~ Cavendish: no electric force inside a hollow conducting shell



- ~ Born-Infeld: vacuum polarization violates superposition at the level of $\alpha^2 = \frac{1}{137^2}$



- ~ linear in both q & Q (superposition)
- ~ central force $\vec{\mathscr{r}} \equiv \vec{r} - \vec{r}\,'$
- ~ inverse square (Gauss') law $\frac{1}{\mathscr{r}^2}$
- ~ units: defined in terms of magnetostatics

$$\varepsilon_0 = 8.85\times10^{-12}\,\frac{C^2}{Nm^2} = \frac{1}{\mu_0 c^2}$$

$$1\,C \equiv 1\,A\cdot s \qquad F/_\ell = 2\times10^{-7}\,N/_m$$

(for parallel wires 1 m apart carrying 1 A each)

- ~ rationalized units to cancel 4π in

$$\nabla\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi\,\delta^3(\vec{\mathscr{r}})$$

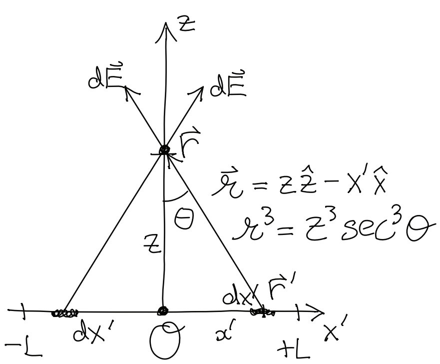
* Electric field
- ~ we want a vector field, but F only at test charge
- ~ action at a distance: the field 'caries' the force from source pt. to field pt.

$$\vec{F} = \underbrace{\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1\hat{\mathscr{r}}_1}{\mathscr{r}_1^2} + \frac{q_2\hat{\mathscr{r}}_2}{\mathscr{r}_2^2} + \dots\right)}Q = Q\vec{E}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i\hat{\mathscr{r}}_i}{\mathscr{r}_i^2} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}\,')d\tau'\hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq'\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$dq' \to q_i = q(\vec{r}_i\,') \text{ or } \lambda(\vec{r}\,')d\ell' \text{ or } \sigma(\vec{r}\,')da' \text{ or } \rho(\vec{r}\,')d\tau'$$

* Example (Griffiths Ex. 2.1)



$$dq' = \lambda dx' = \lambda z\sec^2\theta d\theta$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,2\cdot\int_{x'=0}^{L}\frac{dq'\vec{\mathscr{r}}}{\mathscr{r}^3} = \frac{1}{4\pi\varepsilon_0}\int_0^L\frac{2\lambda dx'\cdot z\hat{z}}{(z^2+x'^2)^{3/2}} + 0\hat{x} \quad \text{(sym)}$$

$$= \hat{z}\,\frac{2\lambda}{4\pi\varepsilon_0 z}\int\frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$

$$= \hat{z}\,\frac{2\lambda}{4\pi\varepsilon_0 z}\underbrace{\sin\theta}_{x'/\mathscr{r}}\Big|_{x'=0}^{L}$$

$$= \hat{z}\,\frac{2\lambda}{4\pi\varepsilon_0 z}\frac{L}{\sqrt{z^2+L^2}}$$

$1+\tan^2\theta = \sec^2\theta$
$x' = z\tan\theta$
$dx' = z\sec^2\theta d\theta$
$\mathscr{r}^3 = (z^2+x'^2)^{3/2}$
$= z^3\sec^3\theta$

as $z\to\infty$ $\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z^2}$ as $L\to\infty$ $\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}$