Section 2.2 - Divergence and Curl of E

* 5 formulations of electrostatics

Coulomb eq. & Superposition

$$\boxed{\vec{E} = \int \frac{đq'\,\hat{\mathscr{r}}}{4\pi\epsilon_0\,\mathscr{r}^2}} \qquad \vec{F} = q\vec{E} \qquad W = qV$$

Helmholtz

Integral field eq's $\quad\xrightleftharpoons{\text{Gauss}}\quad$ Differential field eq's

$$\boxed{\begin{array}{l}\Phi_E = Q/\epsilon_0 \\ \mathcal{E}_E = 0 \end{array}} \text{ (closed regions)} \quad \underset{\text{Stokes}}{\rightleftharpoons} \quad \boxed{\begin{array}{l}\nabla\cdot\vec{E} = \rho/\epsilon_0 \\ \nabla\times\vec{E} = \mathbf{0}\end{array}}$$

$\updownarrow$ $\qquad\qquad\qquad\qquad$ $\updownarrow$ Poincaré

$$\mathcal{E}_E = -\Delta V \quad \xrightleftharpoons{\text{FTVC}} \quad \vec{E} = -\nabla V$$

Potential $\qquad\qquad\qquad$ Poisson eq.

$$\boxed{V = \int \frac{đq'}{4\pi\epsilon_0\,\mathscr{r}}} \quad \underset{\text{Green}}{\overset{\text{Laplace}}{\rightleftharpoons}} \quad \boxed{\nabla^2 V = -\rho/\epsilon_0}$$

~ all of electrostatics comes out of Coulomb's law & superposition principle
~ we use each of the major theorems of vector calculus to rewrite these into five different formulations
- each formulation useful for solving a different kind of problem
~ geometric pictures comes out of schizophrenetic personalities of fields:

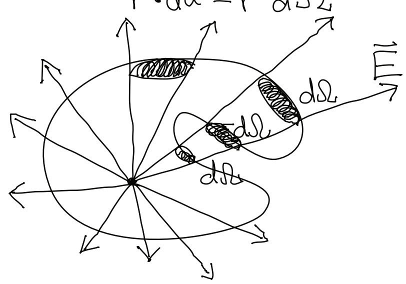
* FLOW (Equipotential surfaces)

$\mathcal{E}_E \equiv \int \vec{E}\cdot d\vec{l}$ ~ integral ALONG the field
~ potential = work / charge

~ $\mathcal{E}_E$ equals # of equipotentials crossed
~ $\Delta\mathcal{E}_E = 0$ along an equipotential surface
~ density of surfaces = field strength

* Gauss' law

$\hat{r}\cdot d\vec{a} = r^2 d\Omega$



~ solid angle

$$d\Omega \equiv \frac{\hat{r}\cdot d\vec{a}}{r^2}$$

~ angle (rad.)

$$d\vec{\theta} = \frac{\hat{r}\times d\vec{l}}{r}$$

~ solid angle of a sphere

$$d\Omega = \sin\theta\, d\theta\, d\phi = -d\cos\theta\, d\phi$$

$$\int\Omega = \int_{\theta=0}^{\pi} -d\cos\theta \cdot \int_{\phi=0}^{2\pi} d\phi = 2\cdot 2\pi = 4\pi$$

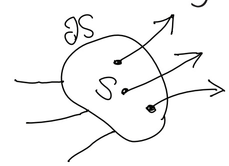
~ $\frac{1}{r^2}$ force laws mean there is a const. flux "carrier" field

* FLUX (Field lines)

$\Phi_E \equiv \int \vec{E}\cdot d\vec{l}$ ~ integral ACROSS the field
~ potential = work / charge

$d\Phi = \vec{E}\cdot d\vec{a}$ = # of lines through area

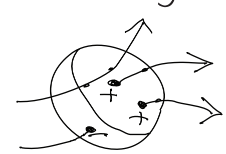
$\vec{E} = \frac{d\Phi}{d\vec{a}}$



~ closed loop

$\int_S d\Phi_E$ = # of lines through loop

~ closed surface



$\oint_S d\Phi_E$ = net # of lines out out of surface
= # of charges inside volume

ϵ_0 is unit of proportionality of flux to charge

* Divergence theorem: relationship between differential and integral forms of Gauss' law

$$\Phi_E = \int_{\partial V} \vec{E}\cdot da = \oint \frac{q\,\hat{\mathscr{r}}}{4\pi\epsilon_0 \mathscr{r}^2}\cdot\hat{\mathscr{r}}\,\mathscr{r}^2 d\Omega = \frac{q}{\epsilon_0} \rightarrow \int_V \frac{dq}{\epsilon_0}$$

$$\int_V \nabla\cdot\vec{E}\, d\tau = \int_V \rho/\epsilon_0\, d\tau$$

~ since this is true for any volume, we can remove the integral from each side

$$\nabla\cdot\vec{E} = \rho/\epsilon_0$$