Section 2.4 - Electrostatic Energy

* analogy with gravity

$\vec{F}=q\vec{E}$	$\vec{F}=m\vec{g}$
$W=q\underbrace{Ed}_{\text{potential}=V}$	$W=m\underbrace{gh}_{\text{potential danger}}$

* energy of a point charge in a potential

$$W=\int_a^b \vec{F}\cdot d\vec{l} = -Q\int_a^b \vec{E}\cdot d\vec{l} = Q\Delta V$$

$$W(\vec{r}) = Q\,V(\vec{r}) \qquad V(\infty)\equiv 0$$

* energy of a distribution of charge $q_1, q_2, \ldots$

$$W=\frac{1}{4\pi\varepsilon_0}\left\{ q_2\frac{q_1}{r_{12}} + q_3\left(\frac{q_1}{r_{13}}+\frac{q_2}{r_{12}}\right) + q_4\left(\frac{q_1}{r_{14}}+\frac{q_2}{r_{24}}+\frac{q_3}{r_{34}}\right)+\ldots\right\}$$

$$=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_iq_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{\substack{i,j=1\\ i\neq j}}^{n}\frac{q_iq_j}{r_{ij}}$$

$$=\frac{1}{2}\sum_{i=1}^{n} q_i \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_j}{r_{ij}} = \frac{1}{2}\sum_{i=1}^{n} q_i V_i(\vec{r}_i) \qquad W=\frac{1}{2}\sum q_iV_i$$

* continuous version

$$\sum_{i=1}^{n} q_i \to \int dq$$

$$W=\frac{1}{2\varepsilon_0}\int \rho\,\nabla^{-2}\rho\, d\tau$$

$$W=\frac{1}{2}\int \rho V d\tau$$

$$W=\frac{\varepsilon_0}{2}\int E^2 d\tau$$

$$\boxed{\frac{dW}{d\tau}=\frac{\varepsilon_0E^2}{2}}$$

* energy density stored in the electric field - integration by parts

$$\nabla\cdot V\vec{E} = \nabla V\cdot\vec{E} + V\nabla\cdot E = -\vec{E}\cdot\vec{E} + V\rho/\varepsilon_0$$

$$0=\int_{\partial\infty} d\vec{a}\cdot(V\vec{E}) = \int_{\infty}\nabla\cdot V\vec{E} = \int -E^2 + V\rho/\varepsilon_0\, d\tau$$

~ is the energy stored in the field, or in the force between the charges?
~ is the field real, or just a calculational device?
~ if a tree falls in the forest ...

* work does work follow the principle of superposition

~ we know that electric force, electric field, and electric potential do

$$\vec{F}=\vec{F}_1+\vec{F}_2 = q(\vec{E}_1+\vec{E}_2+\ ) = -q\nabla(V_1+V_2+\ldots)$$

~ energy is quadratic in the fields, not linear

$$W_{tot}=\frac{\varepsilon_0}{2}\int E^2 d\tau = \frac{\varepsilon_0}{2}\int E_1^2+E_2^2+2\vec{E}_1\cdot\vec{E}_2\, d\tau$$

$$= W_1+W_2+\varepsilon_0\int \vec{E}_1\cdot\vec{E}_2\, d\tau$$

~ the cross term is the 'interaction energy' between two charge distributions (the work required to bring two systems of charge together)