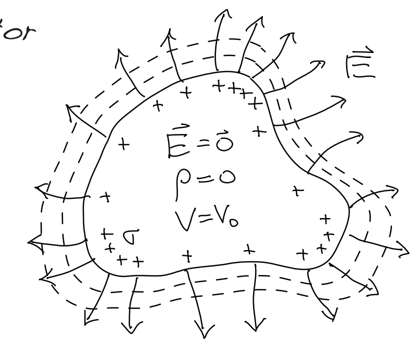
Section 2.5 – Conductors

* conductor
  ~ has abundant "free charge", which can move anywhere in the conductor

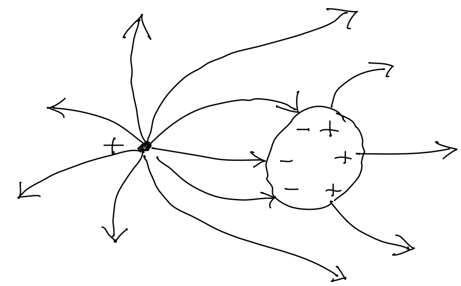
* types of conductors
  i) metal: conduction band electrons, ~ 1 / atom
  ii) electrolyte: positive & negative ions

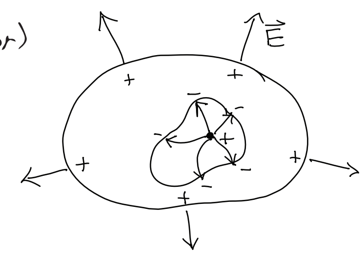
* electrical properties of conductors
  i) electric field = 0 inside conductor
    therefore V = constant inside conductor
  ii) electric charge distributes itself
    all on the boundary of the conductor
  iii) electric field is perpendicular to the
    surface just outside the conductor

	inside	outside
ρ	0	σ
$\vec{E}$	$\vec{0}$	$\frac{\sigma\hat{n}}{\varepsilon_0}$
V	V_0	$V_0+\delta$



* induced charges
  ~ free charge will shift around charge on a conductor
  ~ induces opposite charge on near side of conductor
    to cancel out field lines inside the conductor
  ~ Faraday cage: external field lines are shielded
    inside a hollow conductor
  ~ field lines from charge inside a hollow conductor are
    "communicated" outside the conductor by induction
    (as if the charge were distributed on a solid conductor)
    compare: displacement currents, sec. 7.3





* electrostatic pressure
  ~ on the surface: $\vec{F}/A \equiv \vec{f} = \sigma(\cancel{\vec{E}_{patch}} + \vec{E}_{other}) = \frac{1}{2}\sigma(\vec{E}_{inside} + \vec{E}_{outside})$
  ~ for a conductor: $\vec{E}_{inside}=0 \quad \vec{E}_{out} = \sigma/\varepsilon_0 \qquad P = f = \frac{\sigma^2}{2\varepsilon_0} = \frac{\varepsilon}{2}E^2$
  ~ note: electrostatic pressure corresponds to energy density
    both are part of the stress-energy tensor $\quad P \approx w$

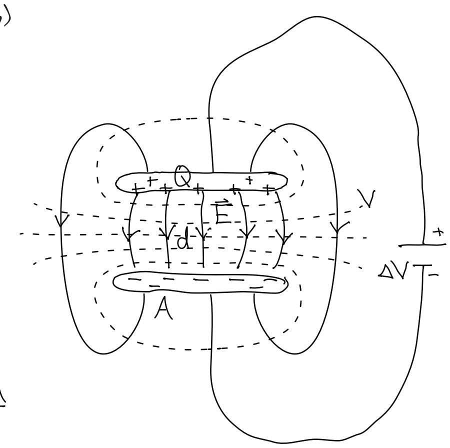
Capacitance

* capacitance
- ~ a capacitor is a pair of conductors held at different potentials, stores charge
- ~ electric FLOW from one conductor to the other equals the POTENTIAL difference
- ~ electric FLUX from one conductor to the other is proportional to the CHARGE

$$C = Q/\Delta V = \frac{\varepsilon_0 \Phi_E}{\mathcal{E}_E} \qquad Q = \int da\, \sigma = \int d\vec{a} \cdot \varepsilon_0 \vec{E} = \varepsilon_0 \Phi_E \quad \text{(closed surface)}$$

$$\Delta V = \int d\vec{l} \cdot \vec{E} = \mathcal{E}_E \quad \text{(open path)}$$

- ~ this pattern repeats itself for many other components: resistors, inductors, reluctance (next sememster)



* work formulation

$$W = \tfrac{1}{2} QV = \tfrac{1}{2} CV^2 = \int \frac{\varepsilon_0}{2} E^2 d\tau$$

$$= \frac{\varepsilon_0}{2} \text{flux} \cdot \text{flow}$$

$$C = \frac{2W}{V^2} = \frac{\varepsilon_0}{V^2} \int E^2 d\tau = \frac{\varepsilon_0}{2} \frac{\text{flux} \cdot \text{flow}}{\text{flow} \cdot \text{flow}}$$

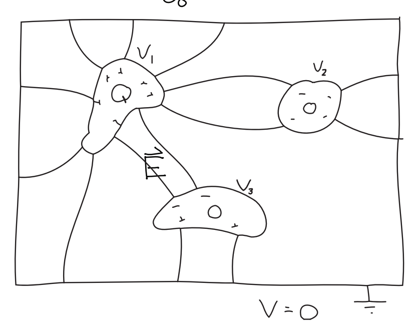
* ex: parallel plates

$$C = \frac{\varepsilon_0 \Phi_E}{\mathcal{E}_E}$$

$$= \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$

* capacitance matrix
- ~ in a system of conductors, each is at a constant potential
- ~ the potential of each conductor is proportional to the individual charge on each of the conductors
- ~ proportionality expressed as a matrix coefficients of potetial P_{ij} or capacitance matrix C_{ij}

$$-\nabla^2 V = \rho/\varepsilon_0 \qquad V(\vec{r}) \propto Q$$



$$V_i = P_{ij} Q_j$$

$$Q_i = C_{ij} V_j$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$$